Overparametrized Linear Regression under Adversarial Attacks

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Uppsala University (Sweden)

SIERRA group meeting
INRIA Paris, March 2023

Outline

The state of Minas Gerais and telehealth

- Minas Gerais

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- More than 4000 ECGs per day

Electrocadiogram exam and machine learning

Goal: *Build data-driven ECG analysis tools.*

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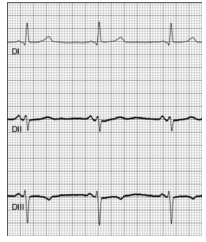

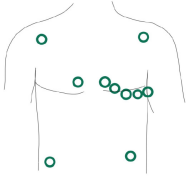

Left: ECG signal **Right:** Electrode placement.

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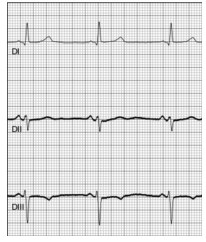

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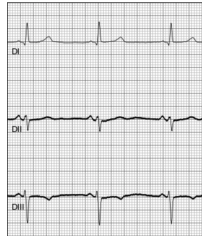

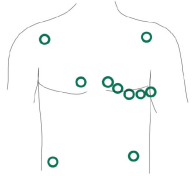

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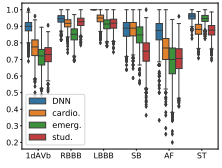

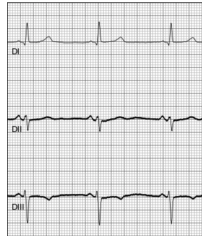

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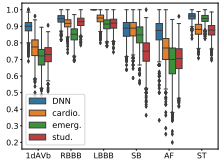

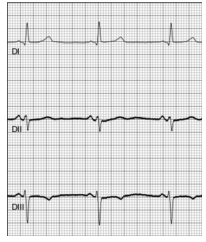

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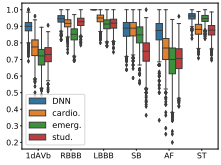

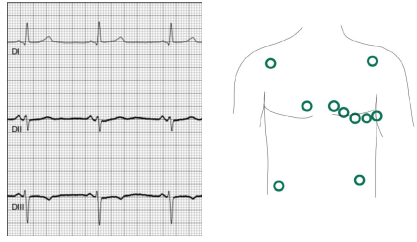

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A. H. Ribeiro , M.H. Ribeiro, Paixão, G.M.M., et al. \Automatic diagnosis of the 12-lead ECG using a deep neural network," Nature Communications, 2020

Adversarial examples

Figure: Effect of adversarial examples on ECG Classification.

Source: Han, X., Hu, Y., Foschini, L. et al. Deep learning models for electrocardiograms are susceptible to adversarial attack. Nature Medicine 26, 360{363 (2020).

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Adversarial robustness

\what is the role of high-dimensionality in model robustness?"

Overparameterized Linear Regression under Adversarial Attacks (2023). **Antônio H. Ribeiro**, Thomas B. Schön. *IEEE Transactions on Signal Processing (preprint: arxiv.org/abs/2204.06274).*

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Adversarial training

\How is it connected to other regularization methods?"'

Surprises in adversarially-trained linear regression (2022). **Antônio H. Ribeiro**, Dave Zachariah, Francis Bach, Thomas B. Schön. *Work in progress.*

Framework: Linear regression

Simplest case where adversarial vulnerability has been observed.

I. J. Goodfellow, J. Shlens, C. Szegedy ,*Explaining and Harnessing Adversarial Examples"*, ICLR 2015
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- Training dataset:

$$(x_1; y_1); (x_2; y_2); \quad ; (x_n; y_n) \ ) \ \text{b}$$

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$$y = b^T x$$

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$$y = b^T x$$

- Error(b) = $y \quad x^T b$
- Adv-error(b) = $\max_k \quad x_k \quad y \quad (x + \quad x)^T b$

Adversarial error in linear regression

- Error(β) = $|y - X^T \beta|$
- Adv-error(β) = $\max_{\|\Delta X\| \le \epsilon} |y - (X + \Delta X)^T \beta|$
- Dual formula for the adversarial error

$$\text{Adv-error}(\beta)^2 = |\text{Error}(\beta)| + \epsilon \|\beta\|_*^2$$

- where $\|\cdot\|_*$ is the dual norm.

ℓ_p-adversarial attacks

- ℓ_1-adversarial attack: $\|x\|_1 \quad g)$ dual norm: $\|x\|_1$

ℓ_p-adversarial attacks

- ℓ_1-adversarial attack: $\delta = \epsilon \cdot \text{sign}(g)$, $\|\delta\|_1 = \epsilon$, dual norm: $\|g\|_\infty$
- ℓ_2-adversarial attack: $\delta = \epsilon \cdot g / \|g\|_2$, $\|\delta\|_2 = \epsilon$, dual norm: $\|g\|_2$

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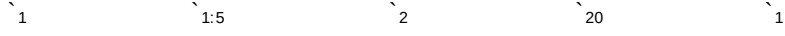
- ℓ_p-adversarial attack: $\delta = \varepsilon \cdot |g|^{q-1} \cdot \mathrm{sign}(g) / \|g\|_q^{q-1}$ dual norm: $\|\cdot\|_q$ for $1/p + 1/q = 1$



Outline

What is the role of high-dimensionality in model robustness?

- High-dimensionality as a source of vulnerability:

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J. Gilmer et al., \Adversarial Spheres," arXiv:1801.02774, Sep. 2018.

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- High-dimensionality as a source of robustness:

S. Bubeck and M. Sellke, \A Universal Law of Robustness via Isoperimetry," Advances in Neural Information Processing Systems, 2021

How neural networks can perform well?

Language models

Image classi cation

Figure: Models number of parameters

Sources: J. Simon (2021) \Large Language Models: A New Moore's Law?". Online (acessed: 2021-11-09). URL: huggingface.co/blog/large-language-models .
M. Tan and Q. V. Le (2019) \E cientNet: Rethinking Model Scaling for Convolutional Neural Networks," ICML

Overparametrized models

Can a model perfectly t the training data and still generalize well?

- Benign over tting

  P. L. Bartlett, P. M. Long, G. Lugosi, and A. Tsigler, \Benign over tting in linear regression," Proceedings of the National Academy of Sciences, vol. 117, no. 48, pp. 30063{30070, Apr. 2020.

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- Example:

Figure: nonlinear ARX mean squared error (MSE).

A. H. Ribeiro , J. N. Hendriks, A. G. Wills, T. B. Schon. \Beyond Occam's Razor in System Identi cation: Double-Descent when Modeling Dynamics". IFAC SYSID 2021. Honorable mention: Young author award

Setup: minimum-norm interpolator

$$\min_{\beta} \|\beta\|_2 \quad \text{subject to} \quad X\beta = y$$

- Gradient descent in linear regression converges to $\beta^{\min}$ ℓ_2.

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Analysing adversarial robustness

From:

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It follows that:

$$\mathbb{E}[\text{Error}(\hat{b})^2] + \epsilon^2\|\hat{b}\|^2 \leq \mathbb{E}[(\text{Adv. error}(\hat{b}))^2] \leq 2\,\mathbb{E}[\text{Error}(\hat{b})^2] + \epsilon^2\|\hat{b}\|^2 \;:$$

Double-descent in the adversarial loss

$$\mathbb{E}[(\ell_2\text{-adv. error}(\hat{\beta}))^2] \;/\; \mathbb{E}[\text{Error}(\hat{\beta})^2] + \; {}^2\kappa\, \|\hat{\beta}\|_2^2 .$$

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$\|\hat{b}\|_2$ also present a double descent behavior. As we increase the problem dimension, it becomes possible to find solutions with smaller norm.

M. Belkin, D. Hsu, S. Ma, and S. Mandal , \Reconciling modern machine-learning practice and the classical bias{variance trade-o ," PNAS (2019)

Asymptotic results

Analysing minimum-norm inteporlation:

$$(x_i ;\ _i) \quad P_x \quad P ; \qquad y_i = x_i^T \ + \ _i ;$$

Figure: Adversarial risk vs number of features m.

T. Hastie, A. Montanari, S. Rosset, and R. J. Tibshirani,\Surprises in High-Dimensional Ridgeless Least Squares Interpolation," Annals of Statisics. 50(2): 949-986 (2022).

High-dimensionality as a source of brittleness

- Model:

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Is robustness at odds with accuracy?

When good model become more vulnerable to adversarial attacks as we add features?

Proposition

If $E[\text{Error}(\hat{b})^2] < \quad :$

$$E[(\text{Adv. error}(\hat{b}))^2] \; ! \; 1 \qquad \text{as} \quad \#\text{features} ! \; 1$$

if and only if

$$k\hat{b}k \; ! \; 1 \quad :$$

Example: optimal predictor vulnerable to adversarial attacks

- Optimal predictor: $\hat{b} = \left[\frac{1}{\#\text{ features}}; \ldots; \frac{1}{\#\text{ features}} \right]$
- $x = E k\hat{x} k_2 / \sqrt{\#\text{ features}}$
- For our example,

$$k\hat{b}k_1 = \sqrt{\#\text{ features}}$$

hence

$$E[(\grave{}_1\text{-adv. error}(\hat{b}))^2] = O(\#\text{ features})$$

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- $x = \mathbb{E}[xx^T] / \sqrt{\#\text{ features}}$
- For our example,

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$$\mathbb{E}[(\ell_1\text{-adv. error}(\hat{b}))^2] = O(\#\text{ features})$$

Mismatched example

- ℓ_1-adv. attack $\|x\|_1$.
- $x / \mathbb{E}[xx^T]$.

:

Example: minimum $\grave{}_2$-norm interpolator

Minimum $\grave{}_2$-norm interpolator and Gaussian features:

$$k^{b}k_1 = O(1) \qquad k^{b}k_2 = O(1=^{p}\overline{m})$$

Now, if we scale

$$/ \quad Ekxk_2 = O(^{p}\overline{m}):$$

Figure: Adv. risk.

The e ect of adversarial training and regularization

Empirical risk minimization:

$$\min \frac{1}{n} \sum_{i=1}^{n} (y_i \quad x_i^T \quad)^2$$

Adversarial training:

$$\min \frac{1}{n} \sum_{i=1}^{n} \max_{k \, x_i k} (y_i \quad (x_i + \quad x_i)^T \quad)^2$$

The e ect of regularization

- Ridge and ℓ_2-adversarial training

$$\|\hat{\beta}\|_1 = O(1)$$

- Lasso, ℓ_1 -adversarial training

$$\|\hat{\beta}\|_1 = O(1/\sqrt{m})$$

The e ect of regularization

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Figure: Adversarial ℓ_1 risk and / $\mathbb{E}\|x\|_2$.

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Adversarial training in linear models

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$$\frac{1}{n}\sum_{i=1}^{n} \max_{k} \max_{x_k} (y_i - (x_i + x)^T)^2$$

Adversarial training in linear models

- Adversarial training,

$$\frac{1}{n}\sum_{i=1}^{n} \max_{\|x_k\| \le k} (y_i (x_i + x)^T)^2$$

can be reformulated as

$$\frac{1}{n}\sum_{i=1}^{n} |y_i x_i^T| + k\|k\|^2$$

Lasso and ℓ_1 -adversarial training

- ℓ_1 -adversarial training:

$$\frac{1}{n}\sum_{i=1}^{n} \left| y_i - x_i^T \beta \right| + \rho \|\beta\|_1^2$$

- Lasso:

$$\frac{1}{n}\sum_{i=1}^{n} \left| y_i - x_i^T \beta \right|^2 + \rho \|\beta\|_1$$

Ridge regression and ℓ_2-adversarial training

- ℓ_2-adversarial training:

$$\frac{1}{n}\sum_{i=1}^{n} \left(y_i - x_i^T \beta + \delta \|\beta\|_2 \right)^2$$

- Ridge:

$$\frac{1}{n}\sum_{i=1}^{n} \left(y_i - x_i^T \beta \right)^2 + \delta \|\beta\|_2^2$$

Diabetes example

ridge regression

ℓ_2-adversarial training

lasso regression

ℓ_1 -adversarial training

Figure: Regularization paths.

Minimum-norm solution

Minimum ℓ_2-norm solution

$$\min \|\beta\|_2 \quad \text{subject to} \quad X\beta = y$$

- Gradient descent in linear regression converges to $\beta_{\min}$ ℓ_2.
- Used to study benign overfitting in:

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Minimum ℓ_1-norm solution

$$\min \|\beta\|_1 \quad \text{subject to} \quad X\beta = y$$

- Basis pursuit: i.e. allow you to recover sparse signals.
- Can also be used to study benign over tting and double descent, i.e.,

  F. Koehler, L. Zhou, D. J. Sutherland, and N. Srebro, \Uniform Convergence of Interpolators: Gaussian Width, Norm Bounds and Benign Over tting," presented at the Advances in Neural Information Processing Systems, 2021

Minimum-norm interpolator and adversarial training

Theorem

Adversarial training is minimized at the minimum norm interpolator

$$\min \|k\| \quad \text{subject to} \quad X = y$$

i 0 < < .

Surprises in adversarially-trained linear regression (2022) António H. Ribeiro , Dave Zachariah, Francis Bach, Thomas B. Schön. Work in progress.

ℓ_2-adversarial training vs ridge regression

Relation to min-norm solution

- (corollary) $\hat{\beta}_{\text{min-}\ell_2}$ is the solution to ℓ_2-adversarial training if $0 < \varepsilon < \varepsilon_0$.
- Ridge $\hat{\beta}_{\text{ridge}}(\lambda) \to \hat{\beta}_{\text{min-}\ell_2}$ as $\lambda \to 0^+$.

Figure: Training MSE vs regularization parameter.

ℓ_1-adversarial training vs Lasso

Relation to min-norm solution

- (corollary) $\hat{\beta}_{\text{min-}\ell_1}$ is the solution to ℓ_1-adversarial training if $0 < \epsilon < \epsilon_0$.
- Lasso $\hat{\beta}_{\text{lasso}}(\lambda) \to \hat{\beta}_{\text{min-}\ell_1}$ as $\lambda \to 0^+$ (LARS algorithm)..

Figure: Training MSE vs regularization parameter.

Discussion

- Distinct behavior from other parameter shrinking methods (overparametrized).

Discussion

- Distinct behavior from other parameter shrinking methods (overparametrized).
- Explanation for abrupt transitions. Let:

$$f_i(\ ) = jy_i \quad x_i^T \quad j + \quad k \quad k_2:$$

and assume $jy_i j = kx_i k_2 = 1$

New interpretation for minimum-norm interpolator

Figure: Threshold *vs* number of features m.

New interpretation for minimum-norm interpolator

Figure: Threshold $\quad$ *vs* number of features m.

- Setup: $y_i = x_i^{>} \quad + \quad _i, x_i \quad N(0; r^2 I_m)$ and $\quad _i \quad N(0; \quad ^2)$
- Increasing the number of features for minimum-norm interpolators increases the maximum disturbance $\quad$ in the corresponding adversarial training problems.

Summary

- Dual formula for the adversarial error:

$$\left(\text{Adv-error}(\widehat{\ })\right)^2 = \left(|\text{Error}(\widehat{\ })| + \ \|\widehat{\ }\| \ \right)^2$$

- Consequences to adversarial robustness
  - Simplify analysis of adversarial robustness:

$$\mathsf{E}\left[\left(\text{Adv-error}(\widehat{\ })\right)^2\right] \propto \mathsf{E}\left[\text{Error}(\widehat{\ })^2\right] + \ \|\widehat{\ }\|^2$$

  - Double descent can be observed in adversarial scenarios.
  - Sufficient and necessary conditions for good models to be vulnerable to adversary.
- Consequences to adversarial training:
  - Convex formula / Similarities with parameter shrink methods
  - ℓ_1-adversarial training $\Rightarrow$ sparse solutions
  - Can interpolate for disturbance bounded by $\quad > 0$.

Thank you!

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