Shooting Methods for Parameter Estimation of Output Error Models

Antônio H. Ribeiro[1] and Luis A. Aguirre[2]

[1]Graduate Program in Electrical Engineering UFMG; [2] Department of Electronic Engineering UFMG

System Identification

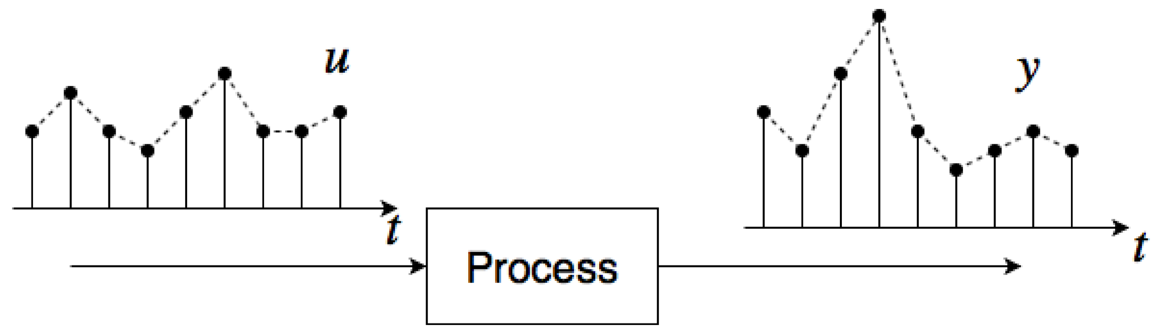

Figure 1: System identification problem.

Prediction Error Methods

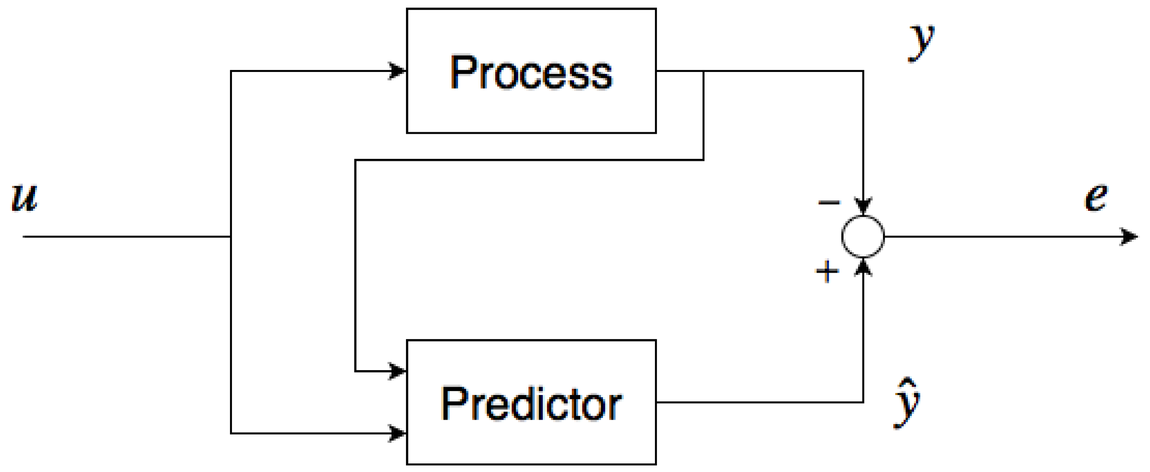

Figure 2: Prediction error methods.

NARX

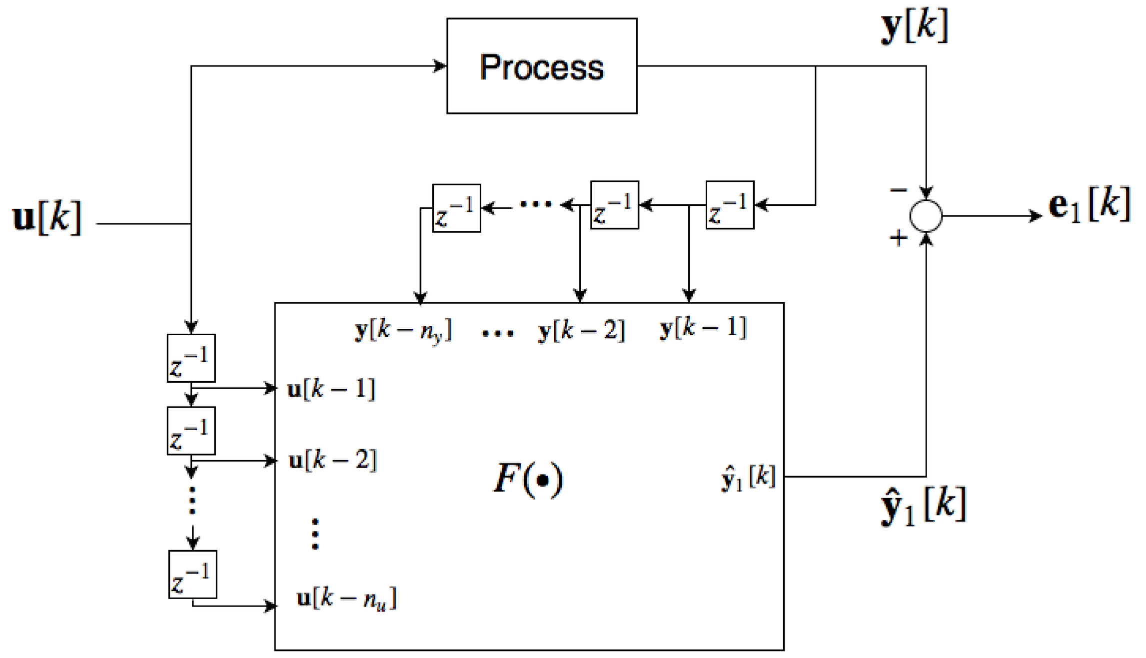

Figure 3: Nonlinear autoregressive with exogenous input (NARX) model.

NOE

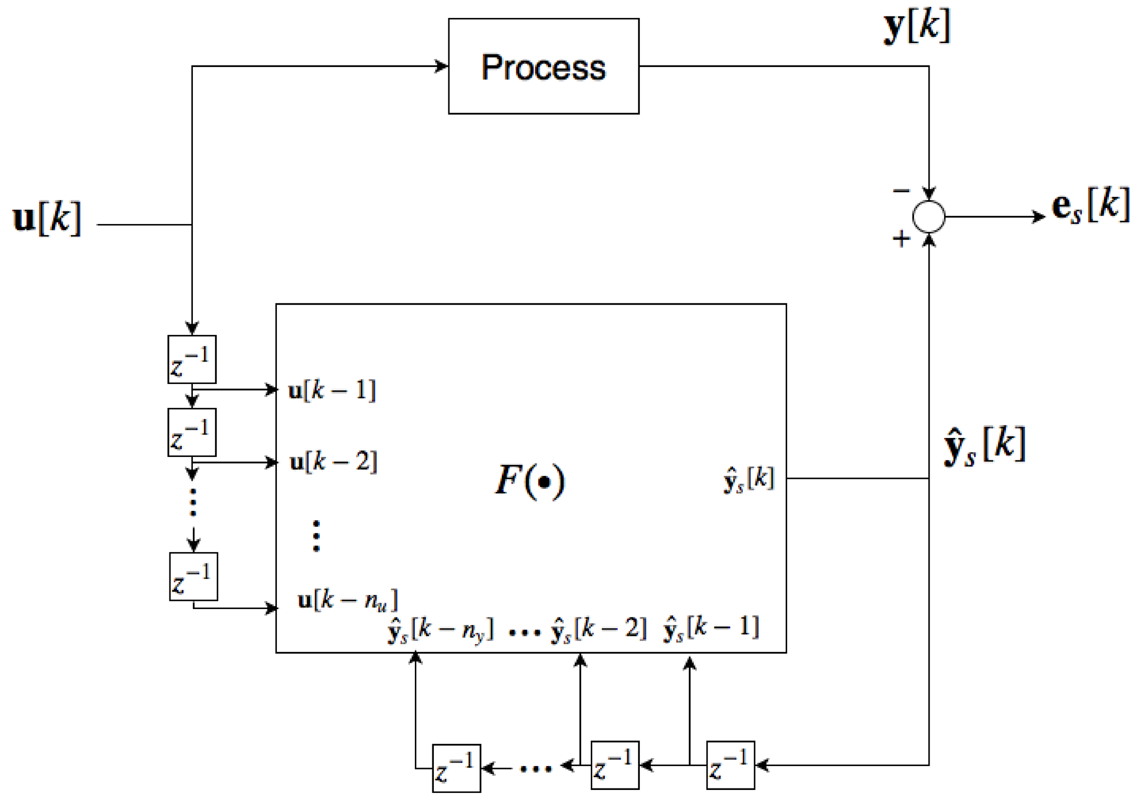

Figure 4: Nonlinear output error (NOE) model.

Possible Advantages of NOE models

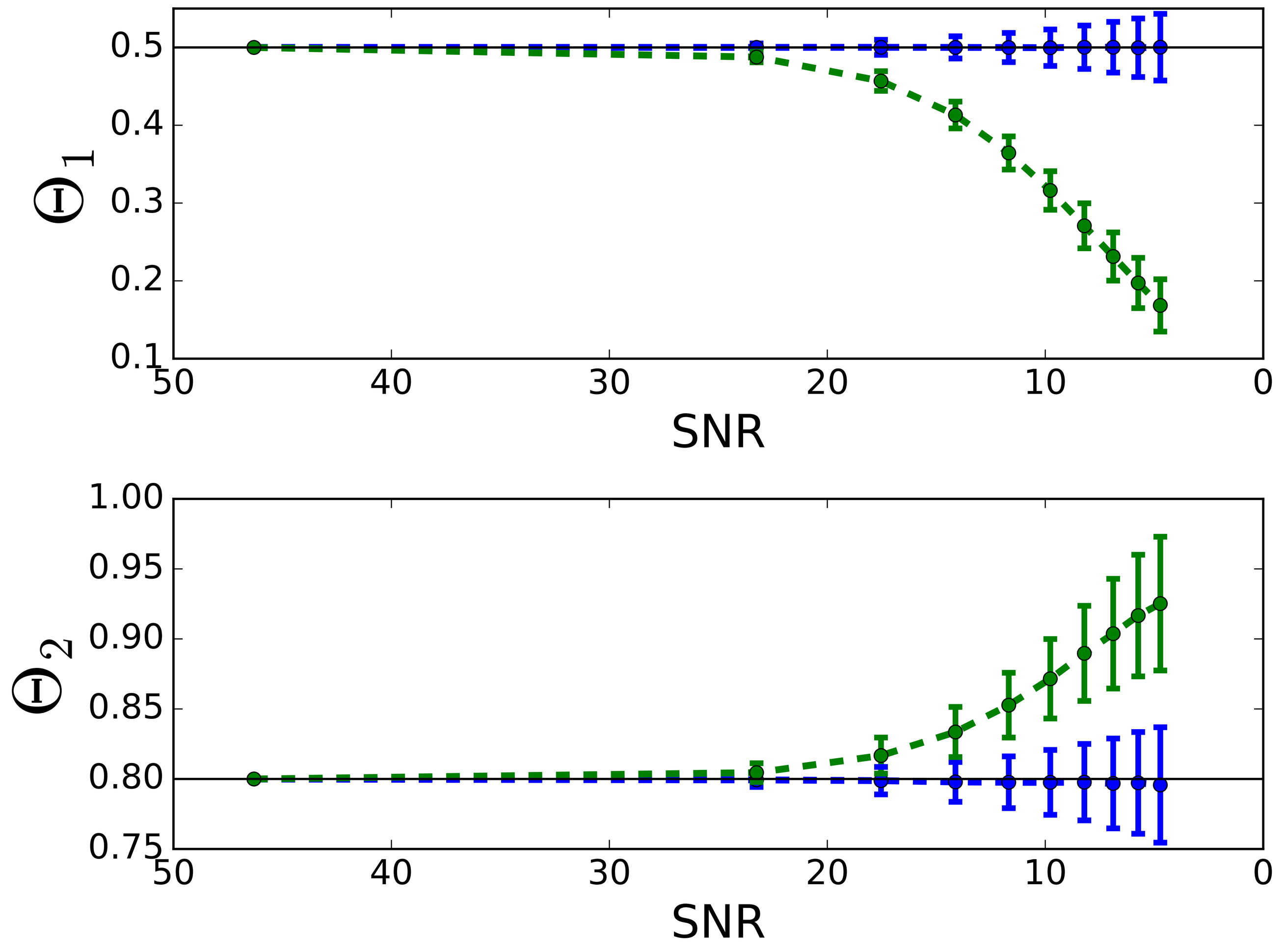

Figure 5: Expected values (dots) and standard deviation $\pm\sigma_{\hat{\Theta}}$ (bars) of parameter. Estimated using an NOE model (in blue) and an NARX model (in green). The true value is represented as the horizontal line.

Multiple Shooting

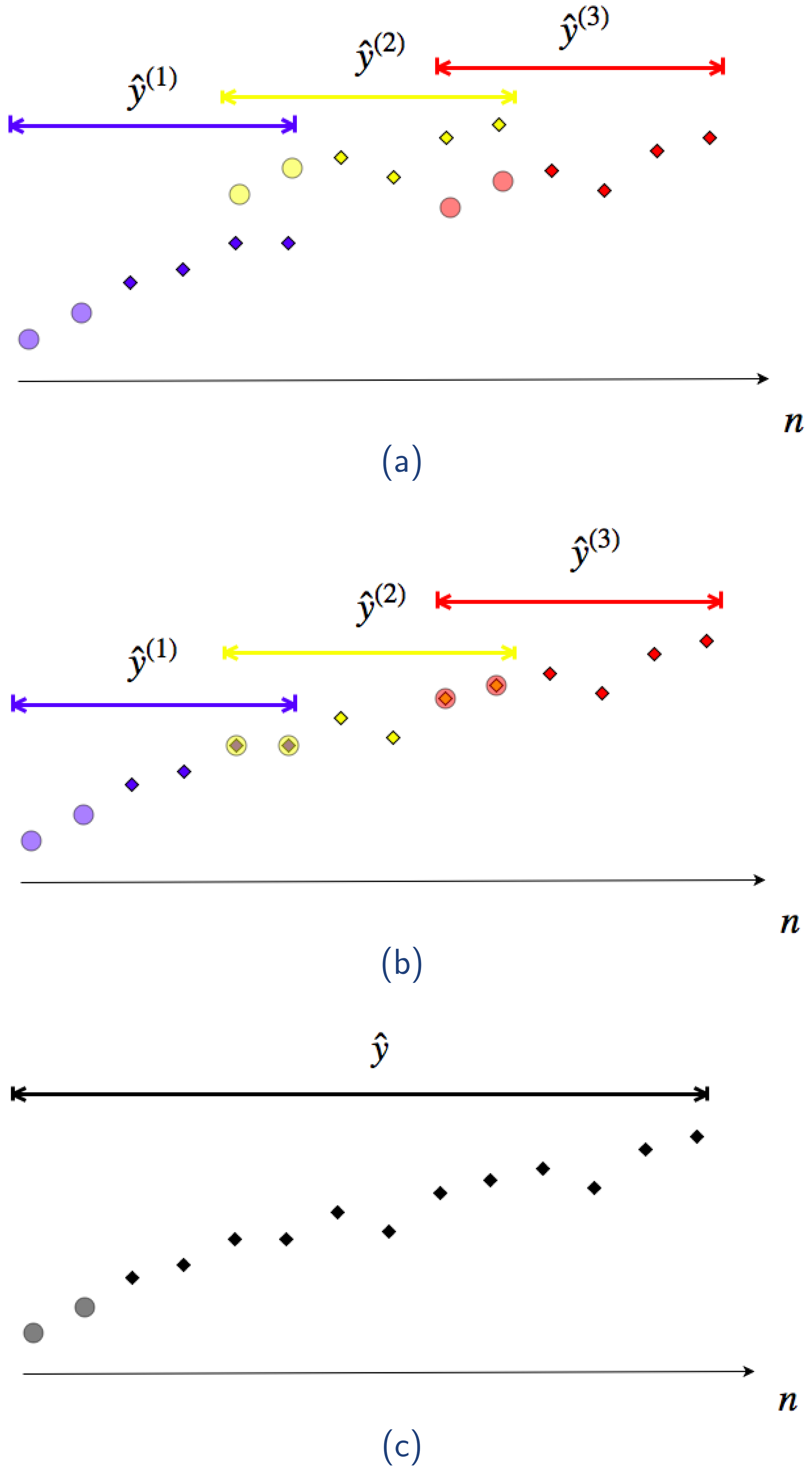

Figure 6: Three consecutive simulations $\hat{y}^{(i)},\ i = 1, 2, 3$ are indicated with different colors. The initial conditions are represented with circles ○ and subsequent simulated values with diamonds ⋄.

Multiple Minima Problem

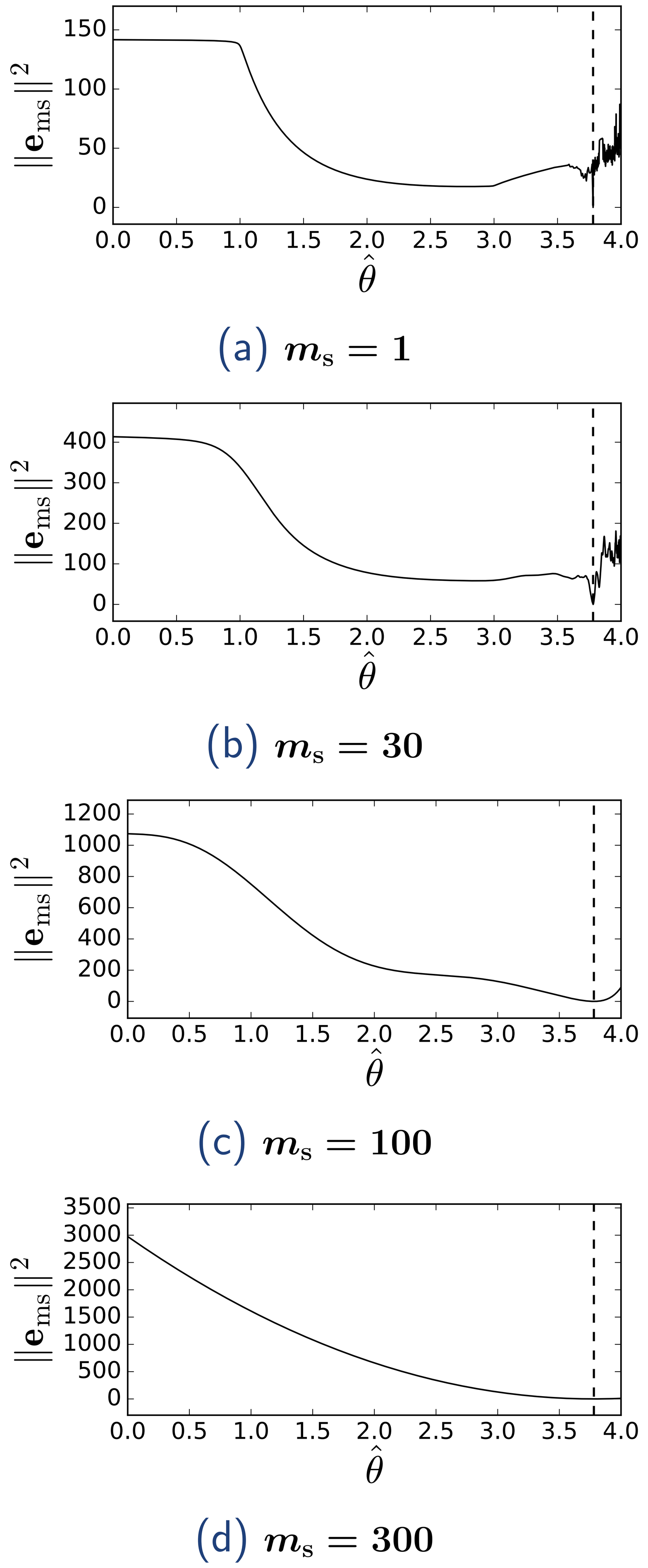

Figure 7: Shows $\|\mathbf{e}_{ms}\|^2$ as a function of $\hat{\theta}$ for fixed initial conditions. The vertical dashed line (— —) represents the true parameter value.

Acknowledges

This work has been supported by the Brazilian agencies CAPES, CNPq and FAPEMIG.

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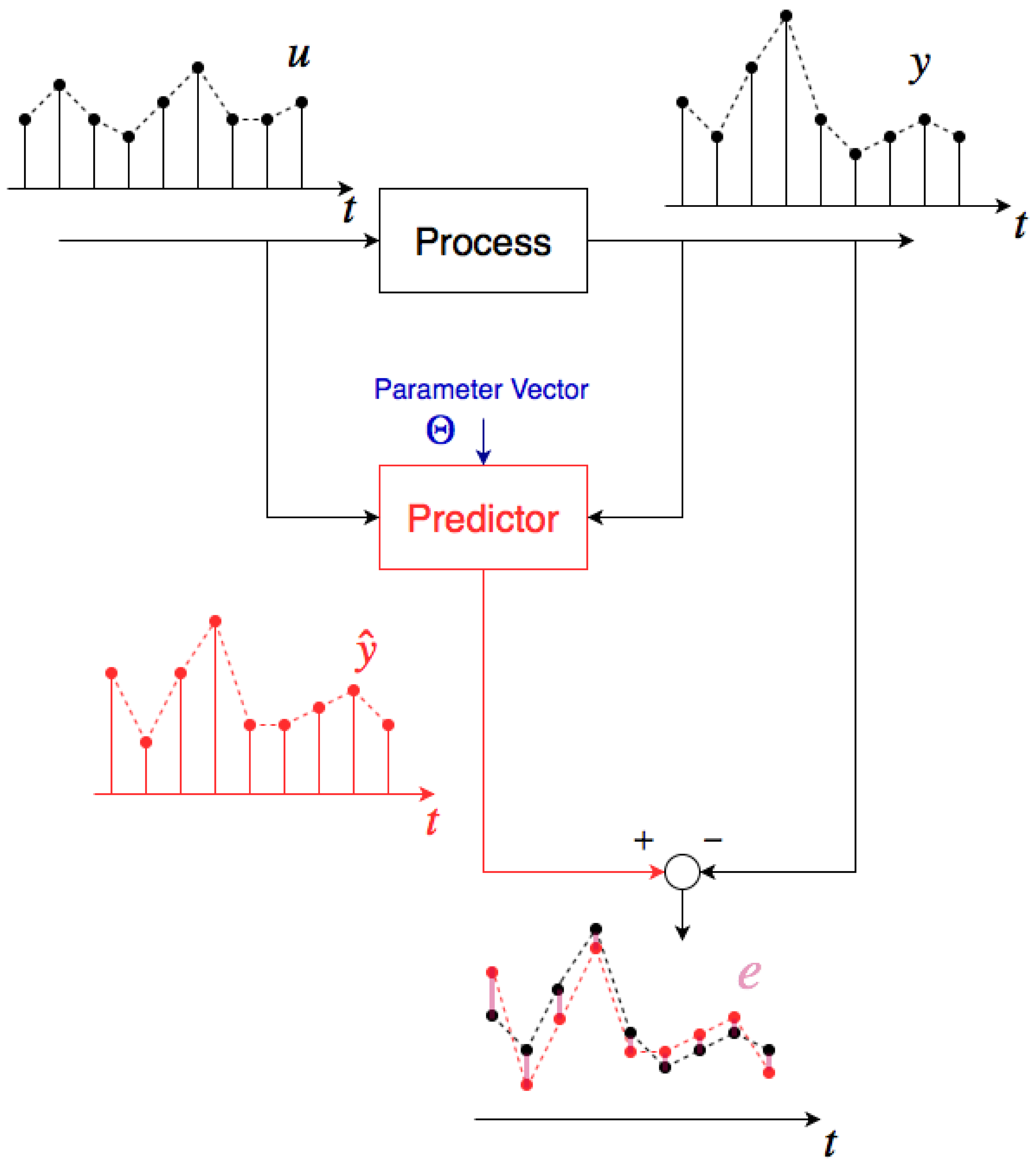

Figure 8: *Prediction error methods.* These methods estimate the parameters by minimizing the error between the optimal output prediction and the measured value

Difference Equation Models

Difference equations:

$$\mathbf{y}[k] = \mathbf{F}\left(\mathbf{y}[k-1], \mathbf{y}[k-2], \mathbf{y}[k-3], \mathbf{u}[k-1], \mathbf{u}[k-2], \mathbf{u}[k-3]; \Theta\right).$$

One-step-ahead Prediction

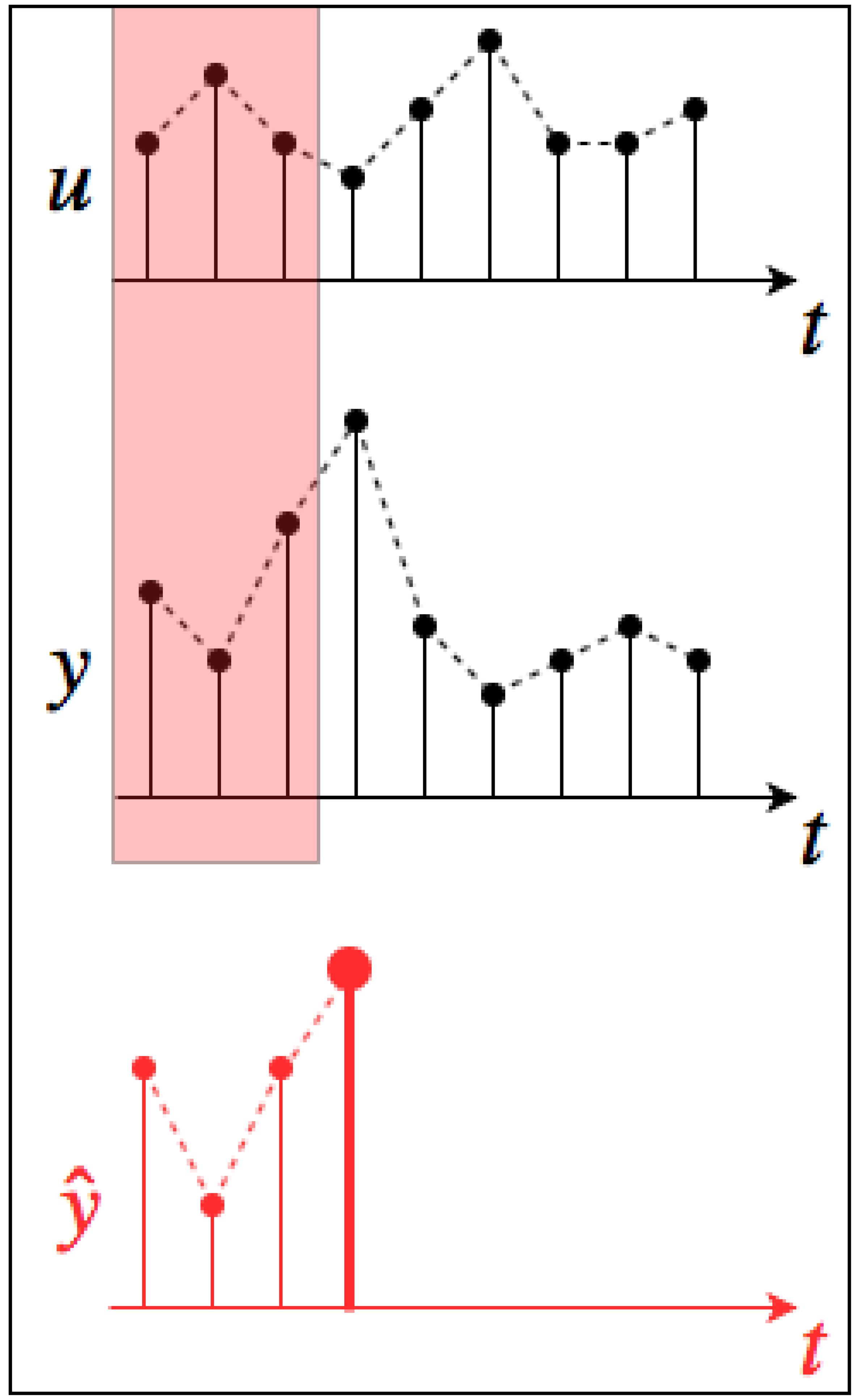

Figure 9: One-step-ahead prediction.

Free-run Simulation

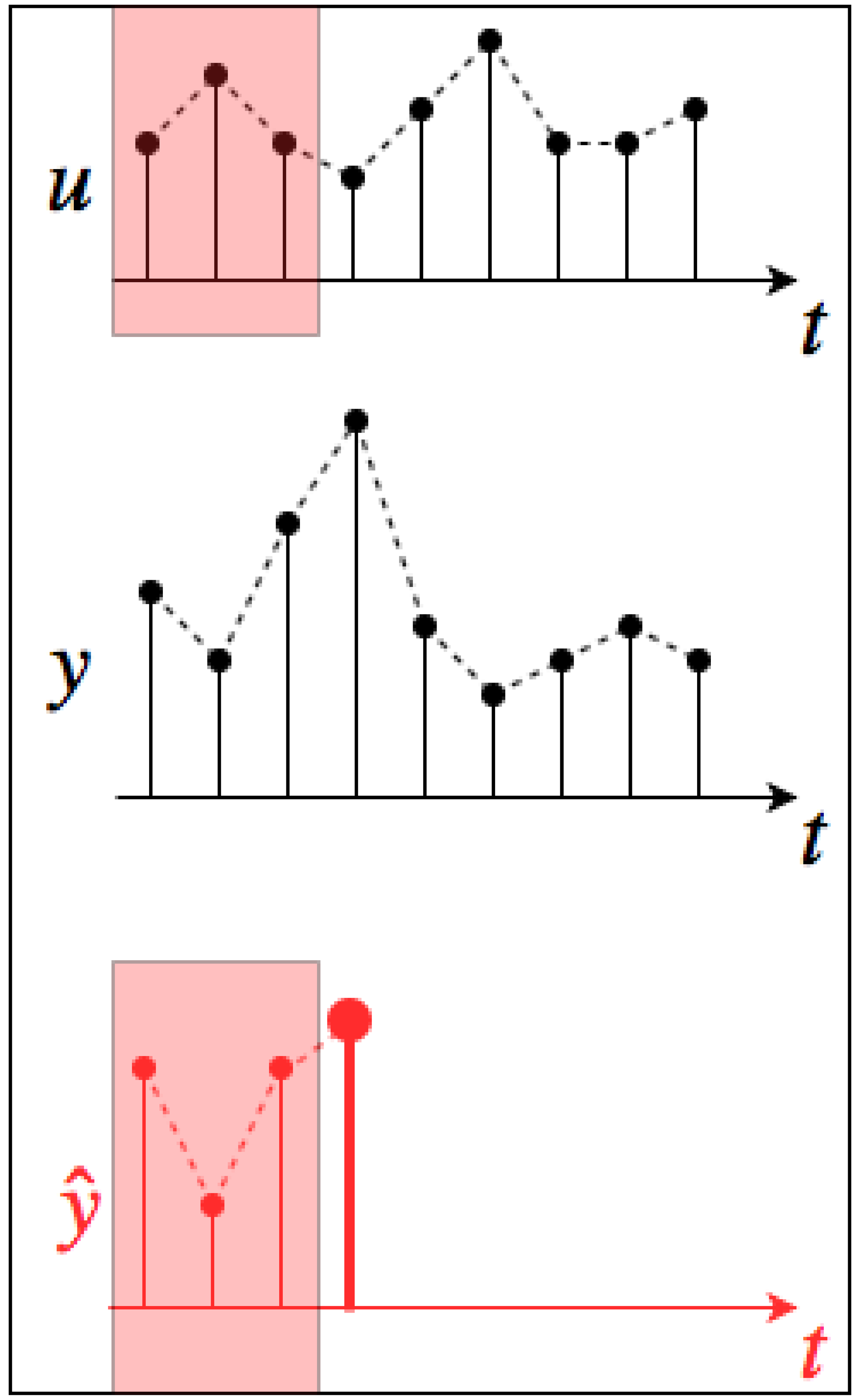

Figure 10: Free-run simulation.

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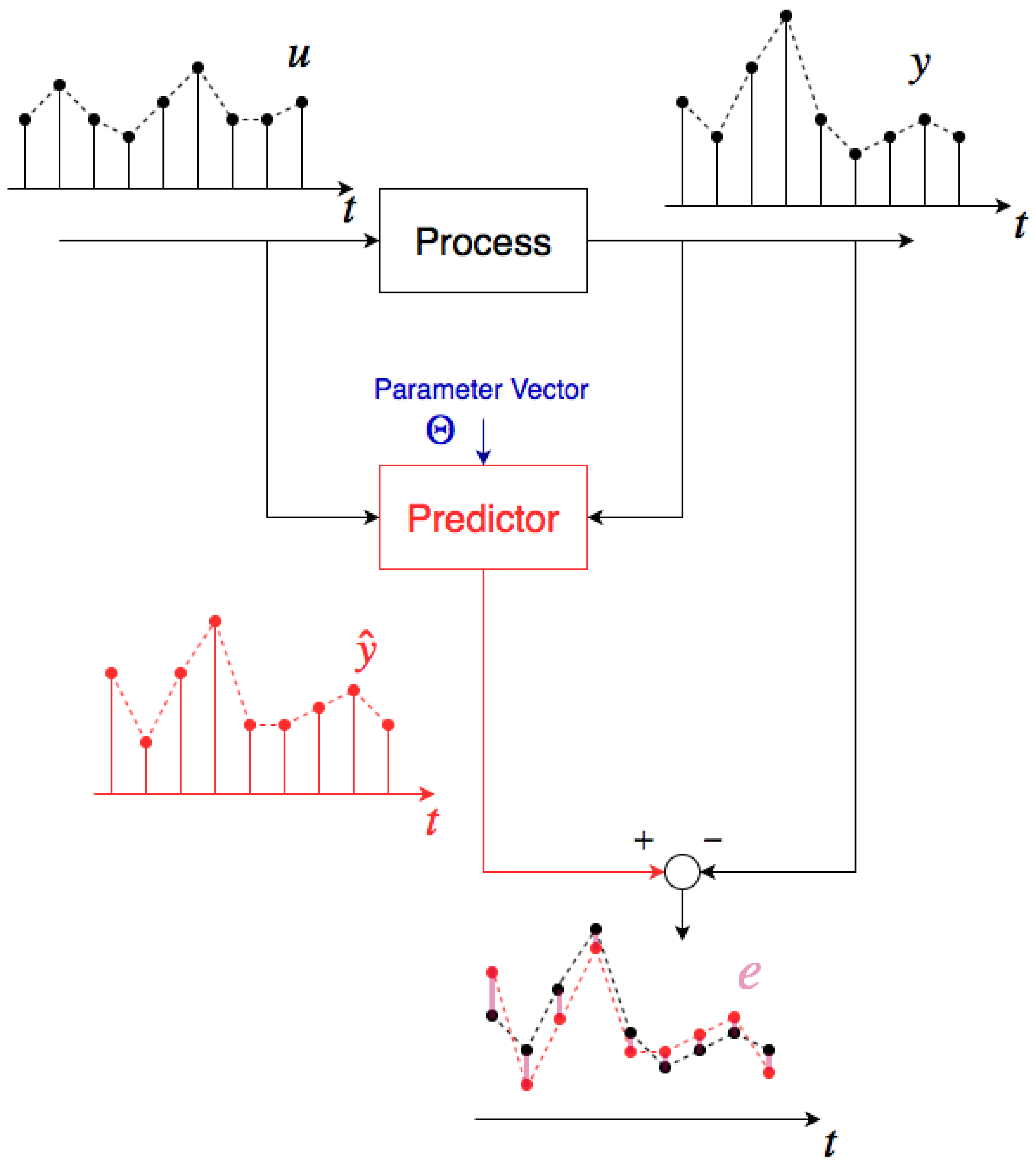

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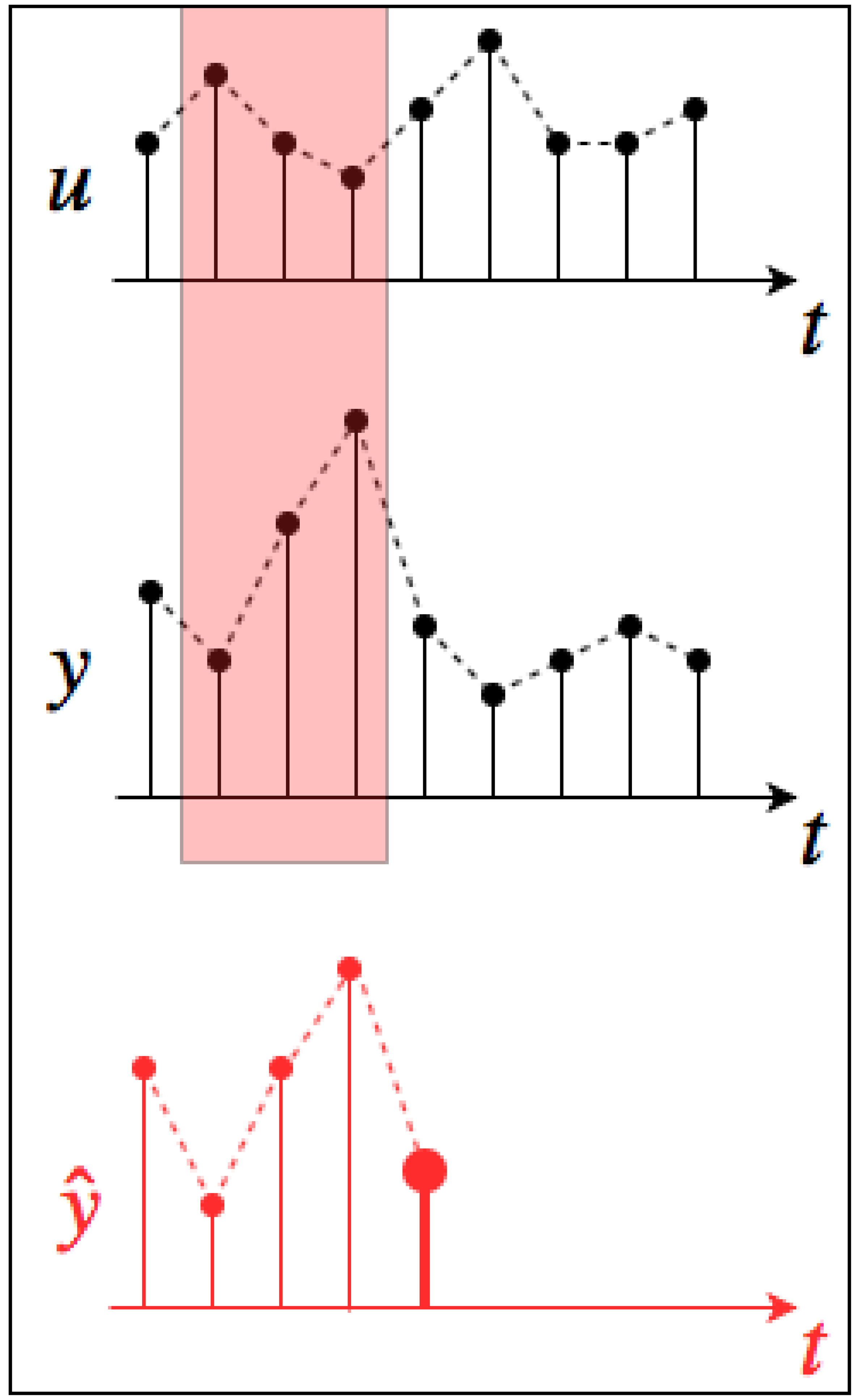

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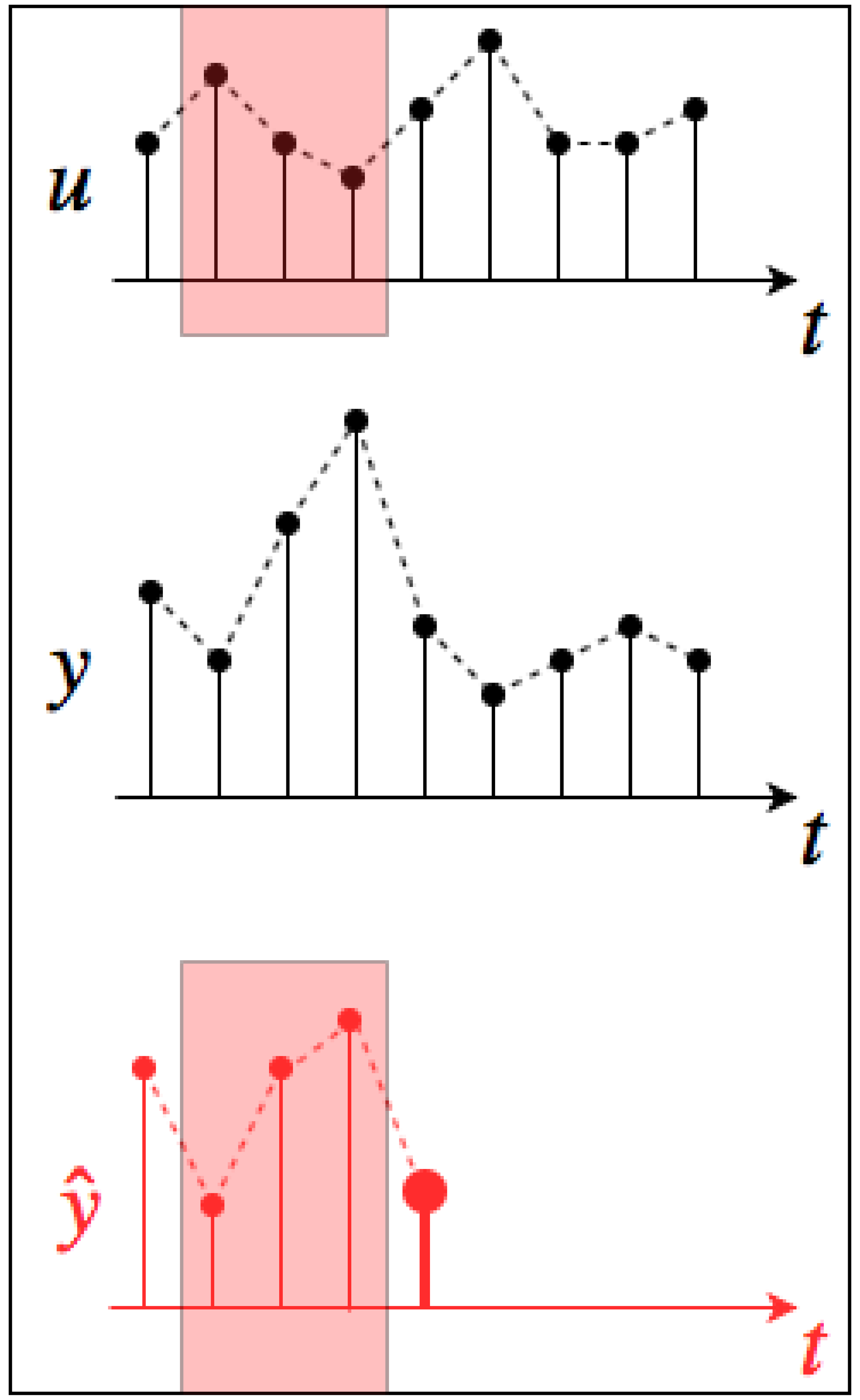

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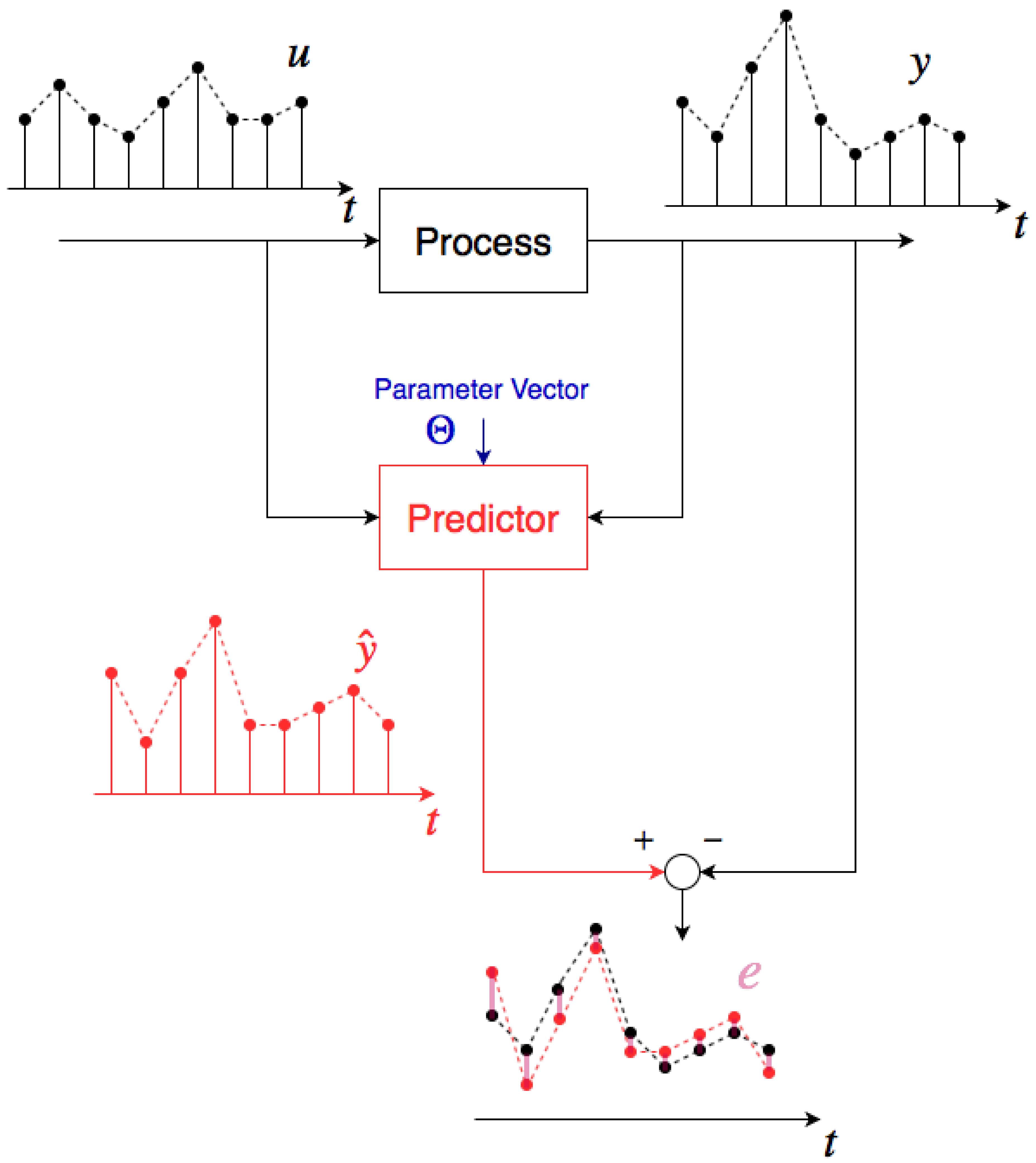

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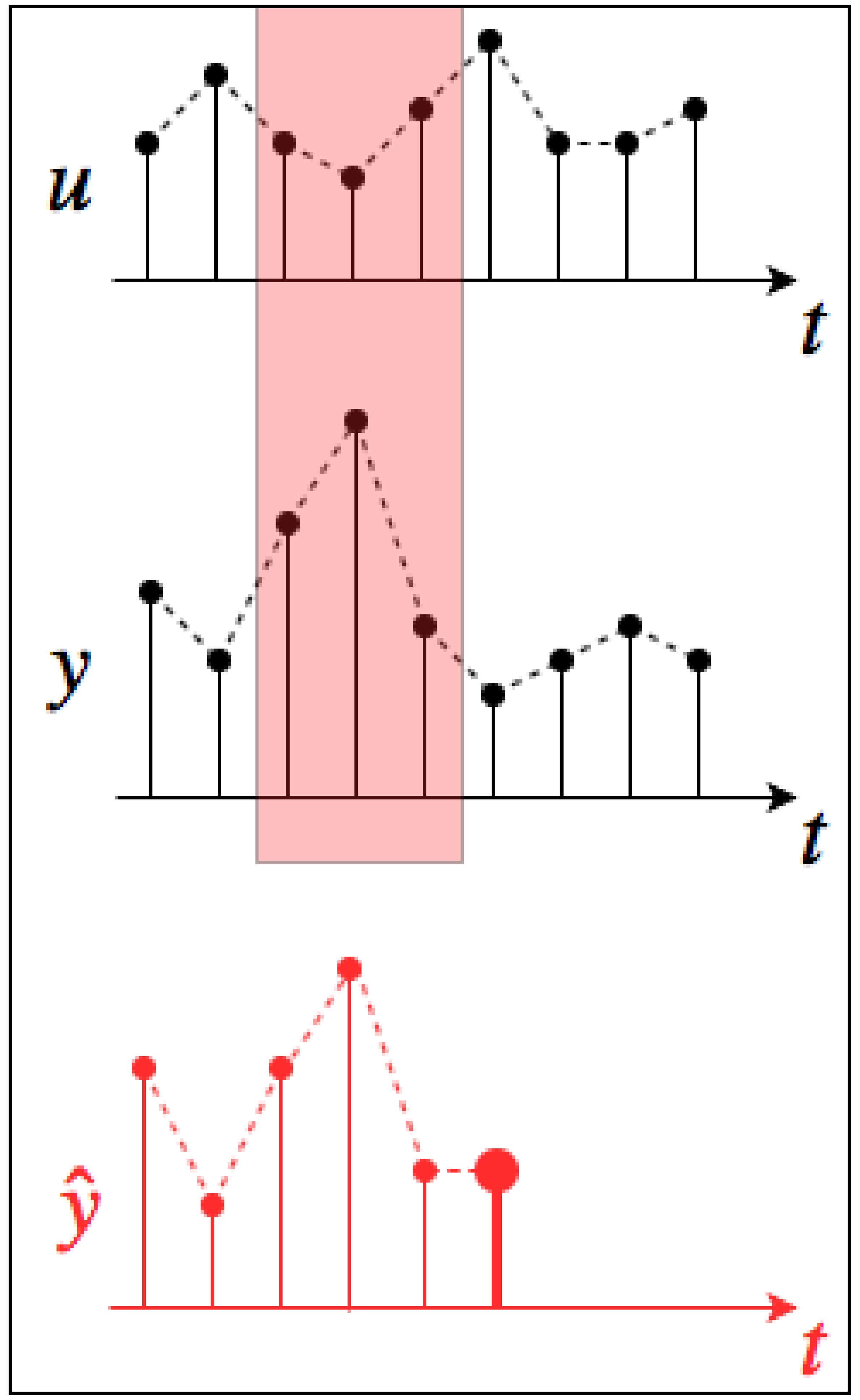

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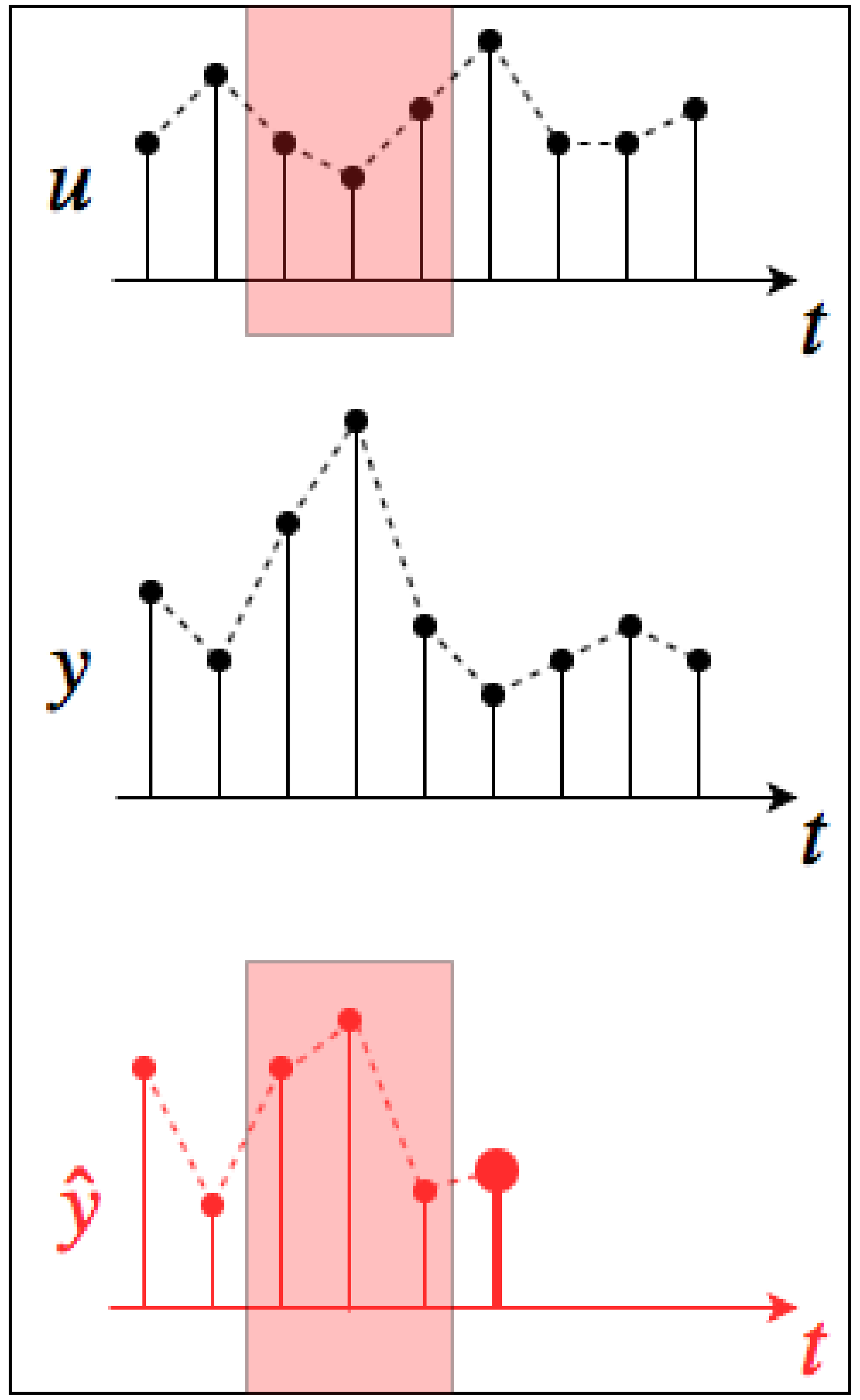

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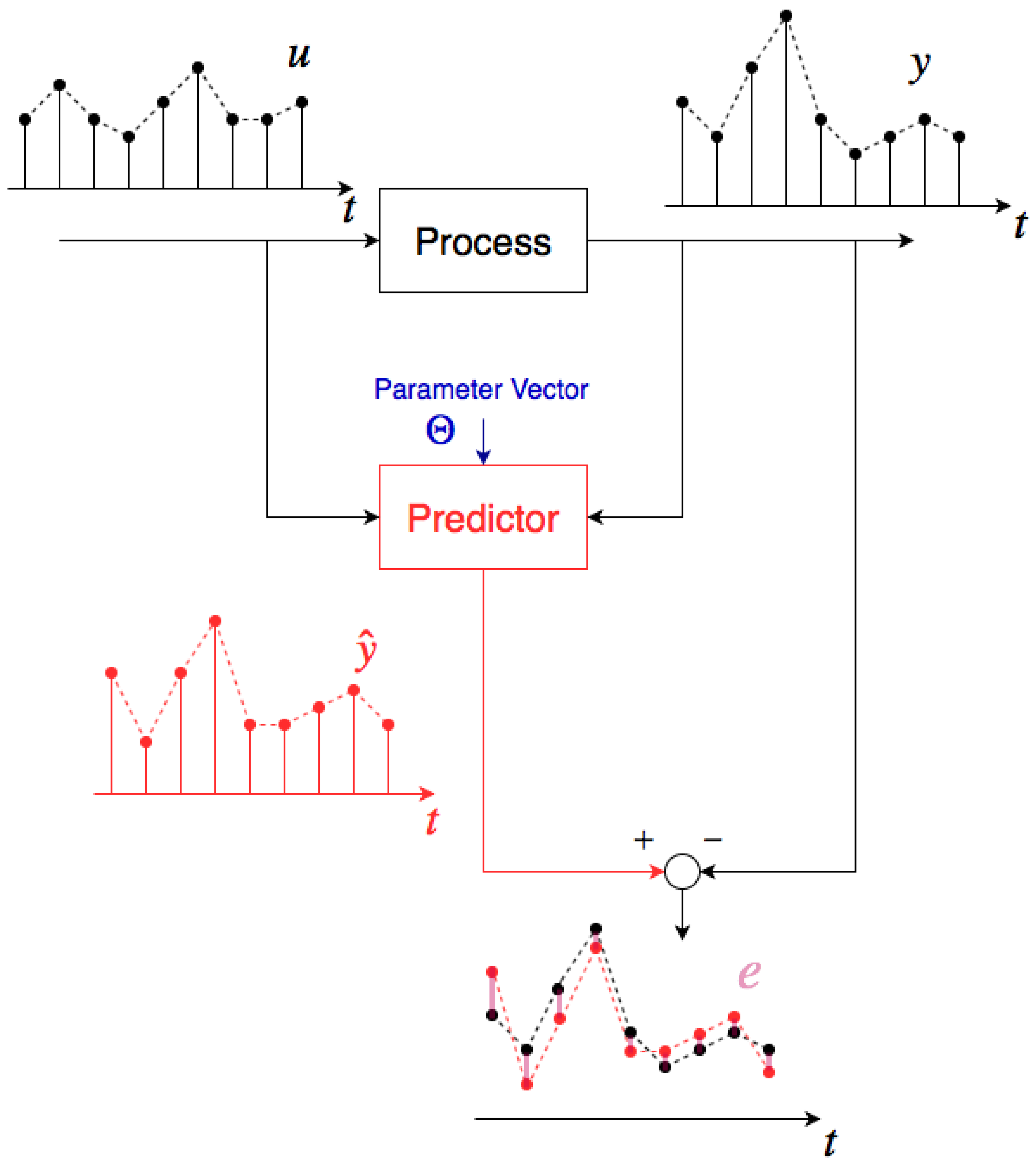

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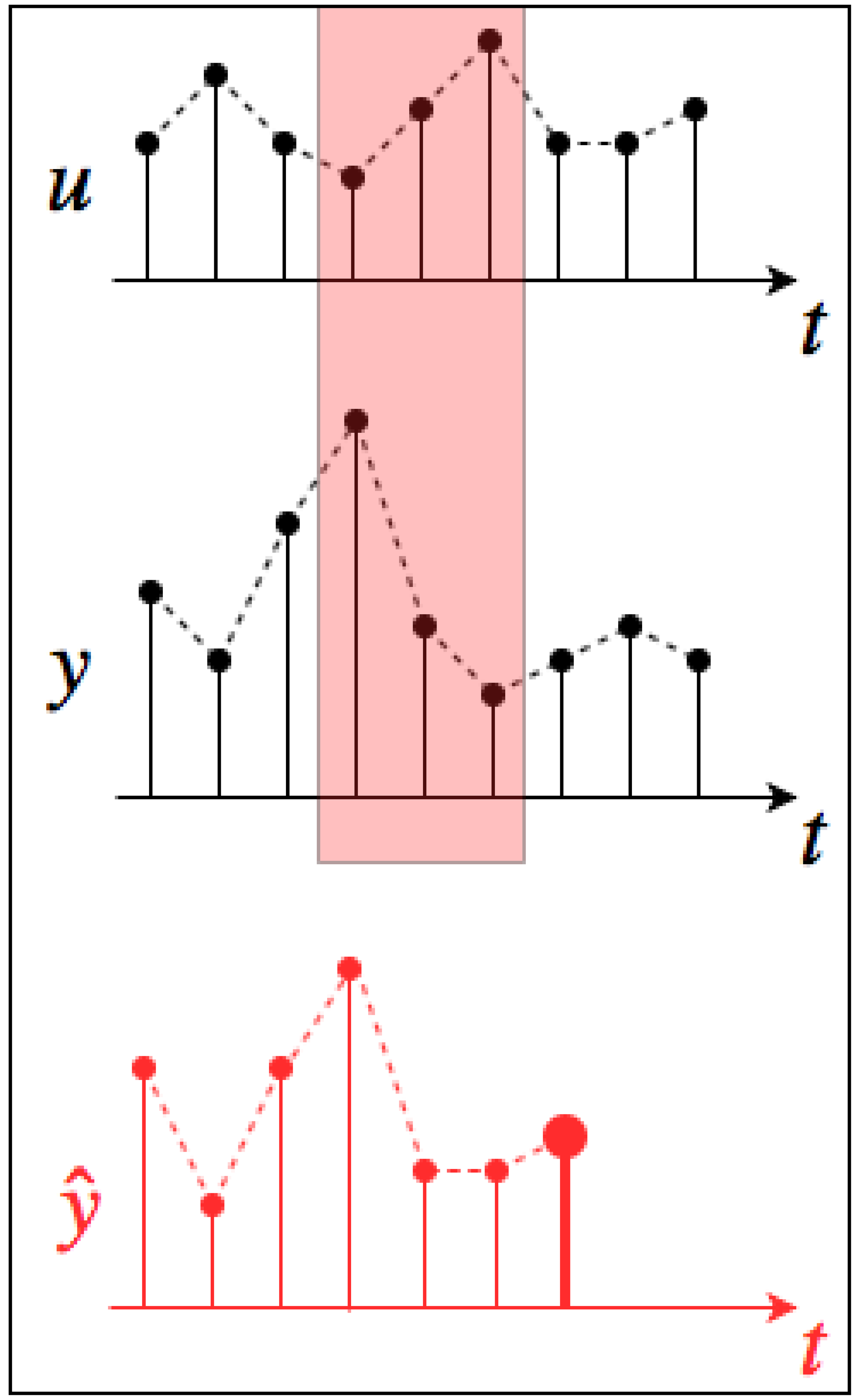

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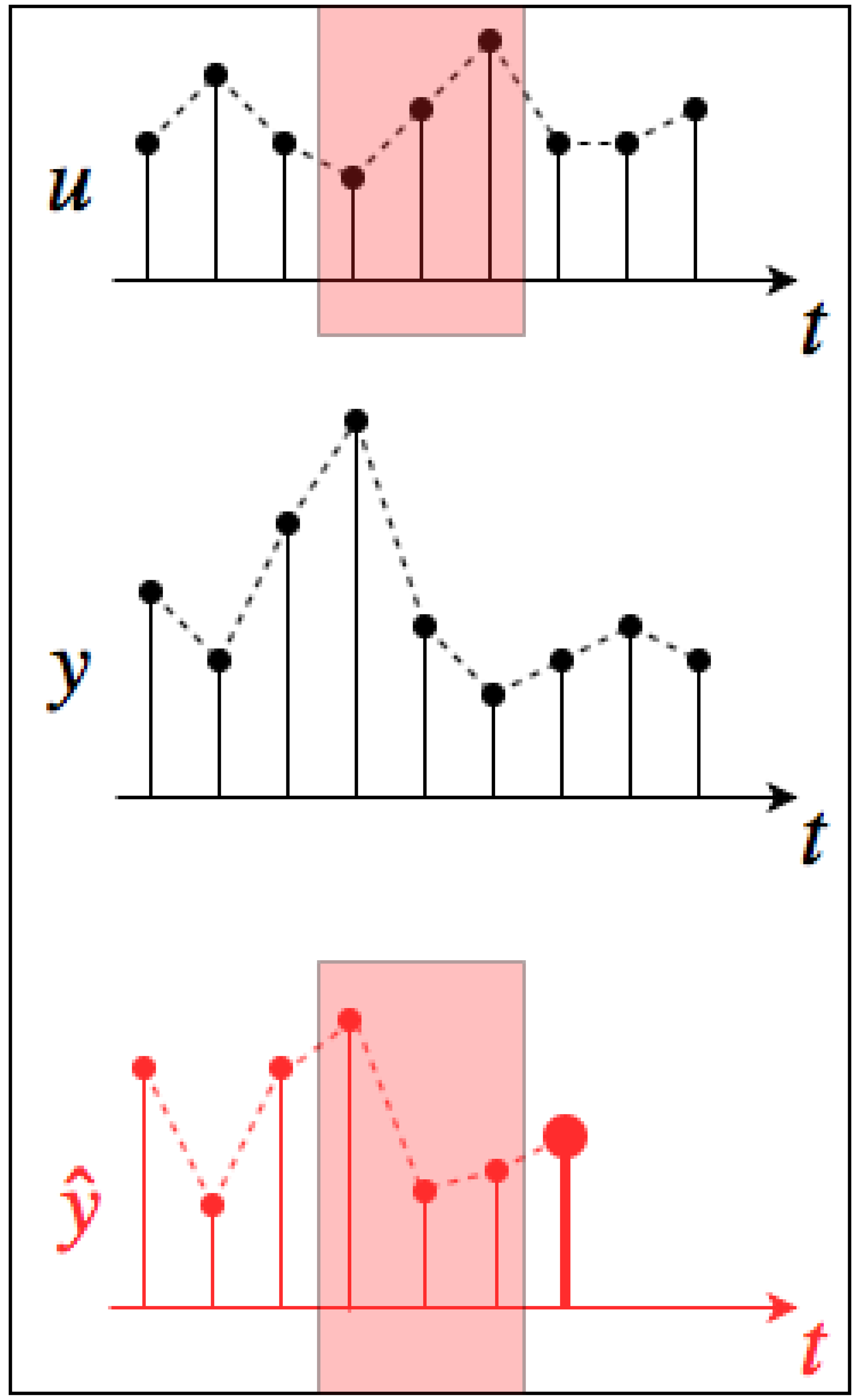

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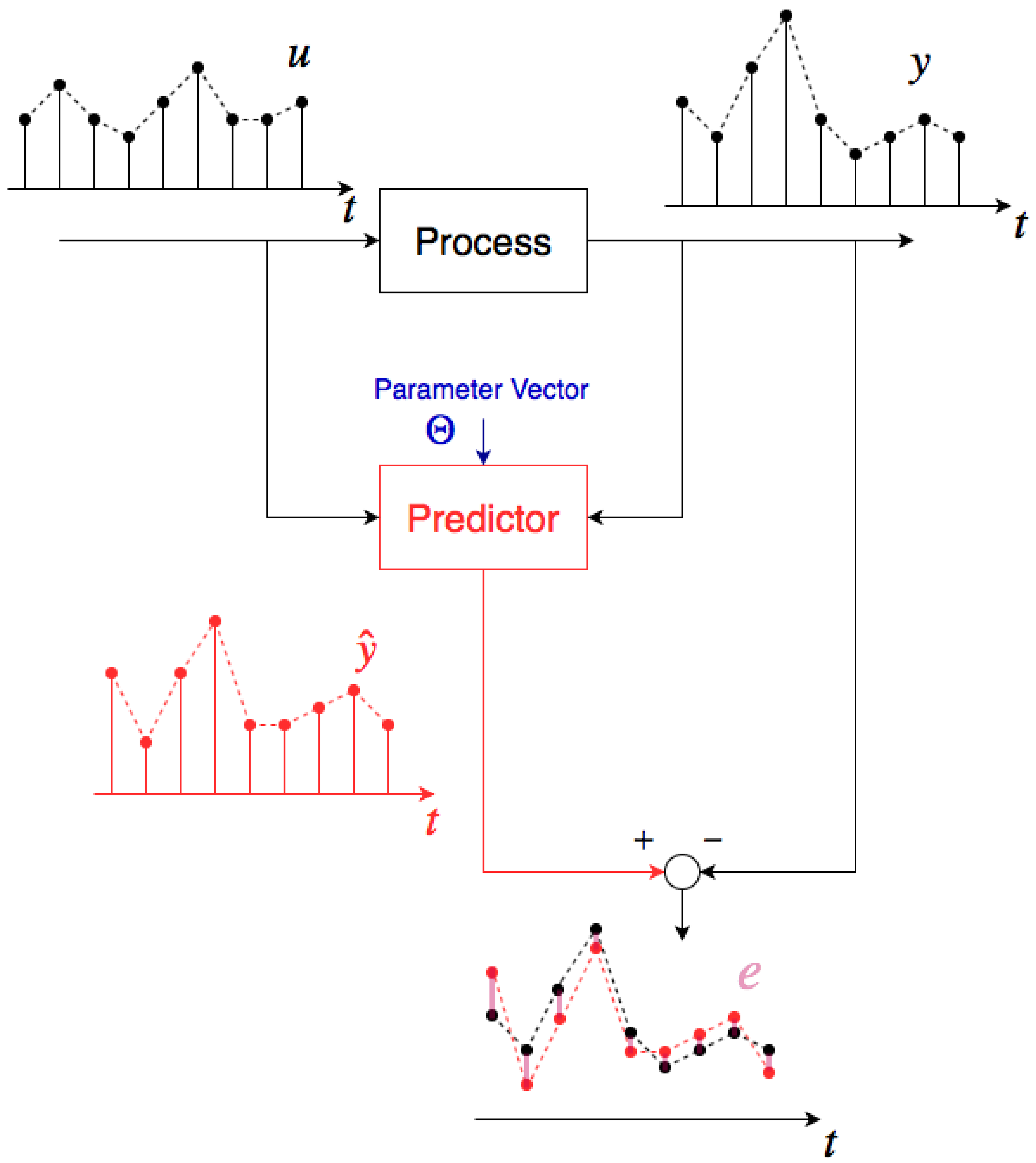

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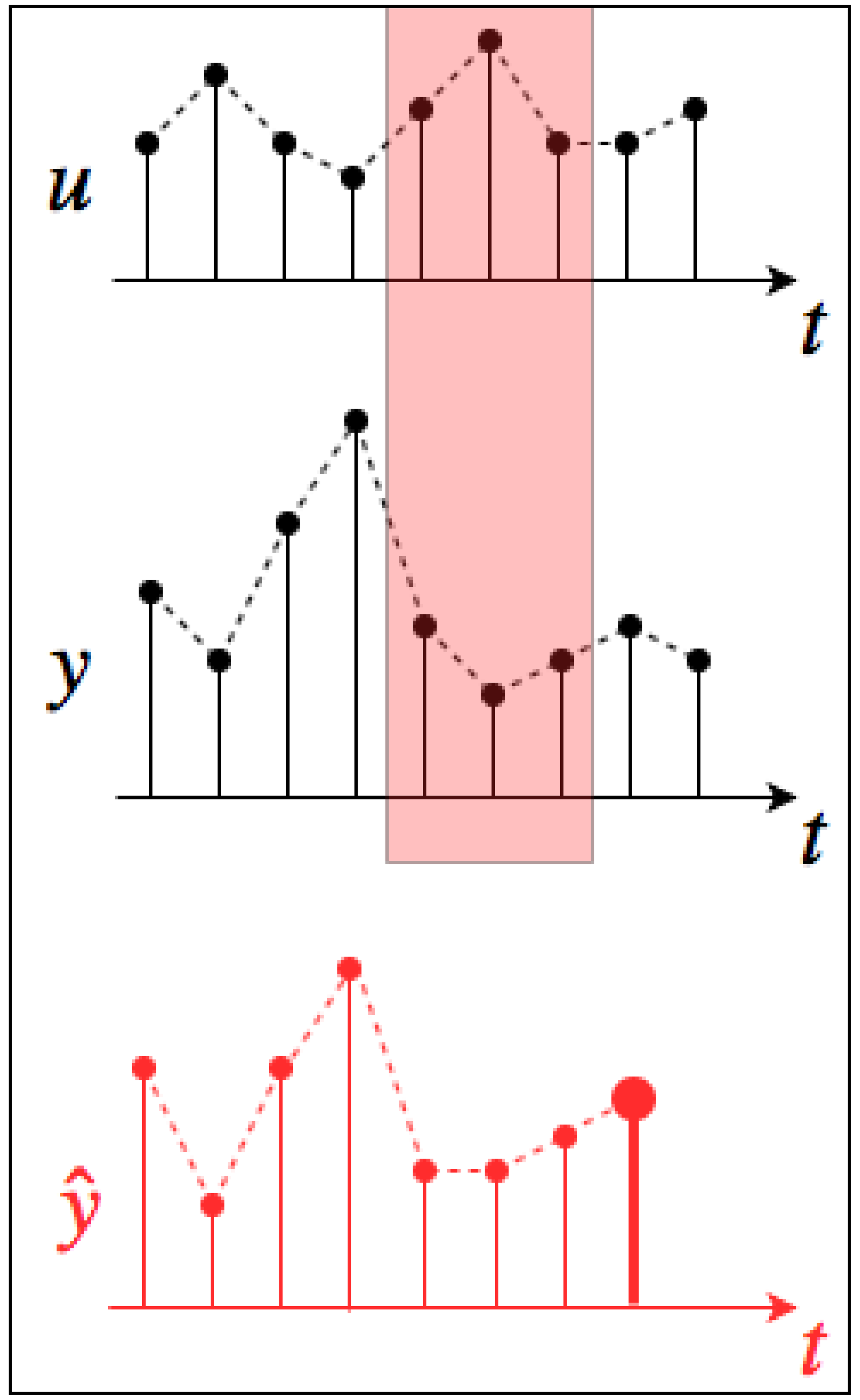

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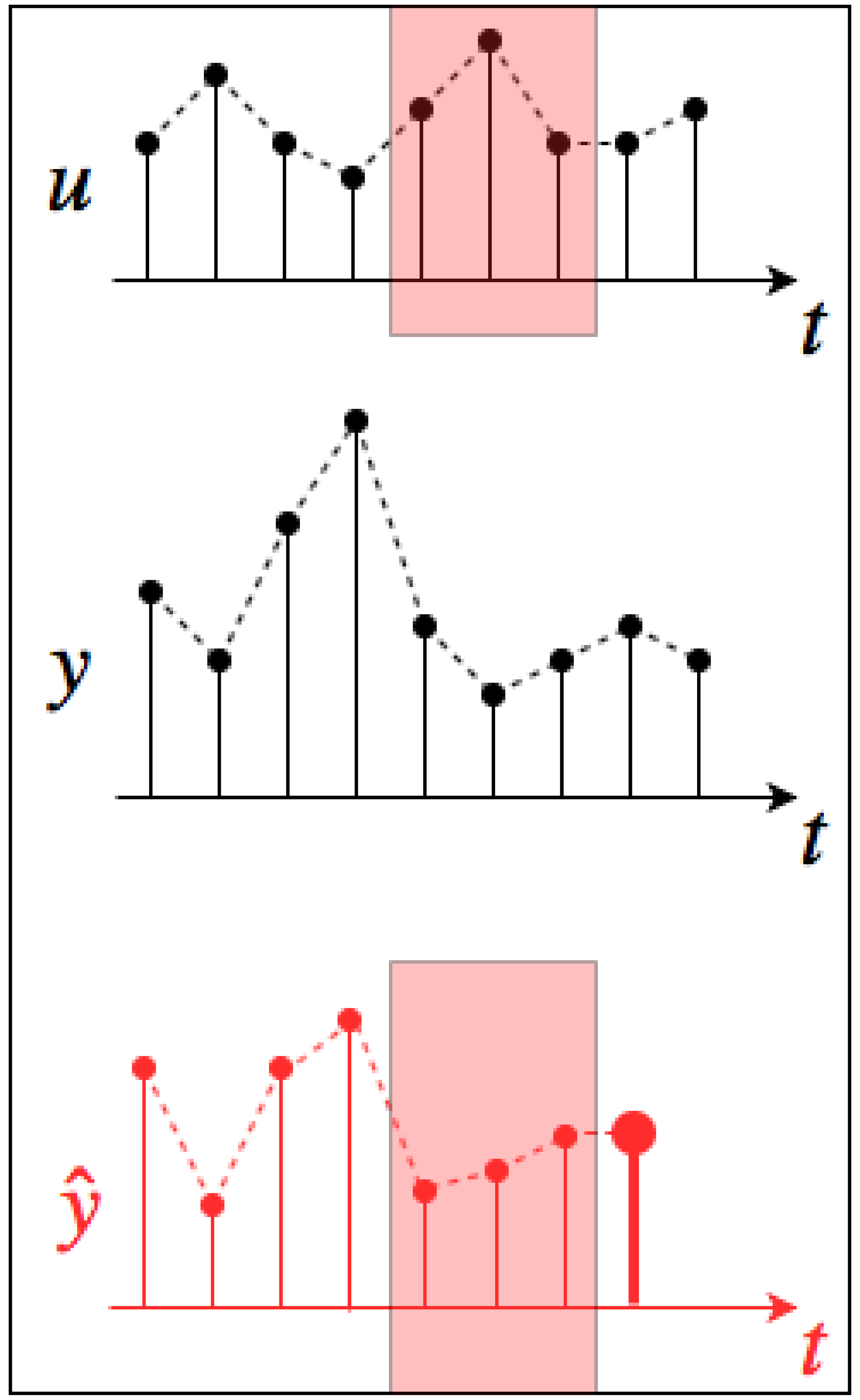

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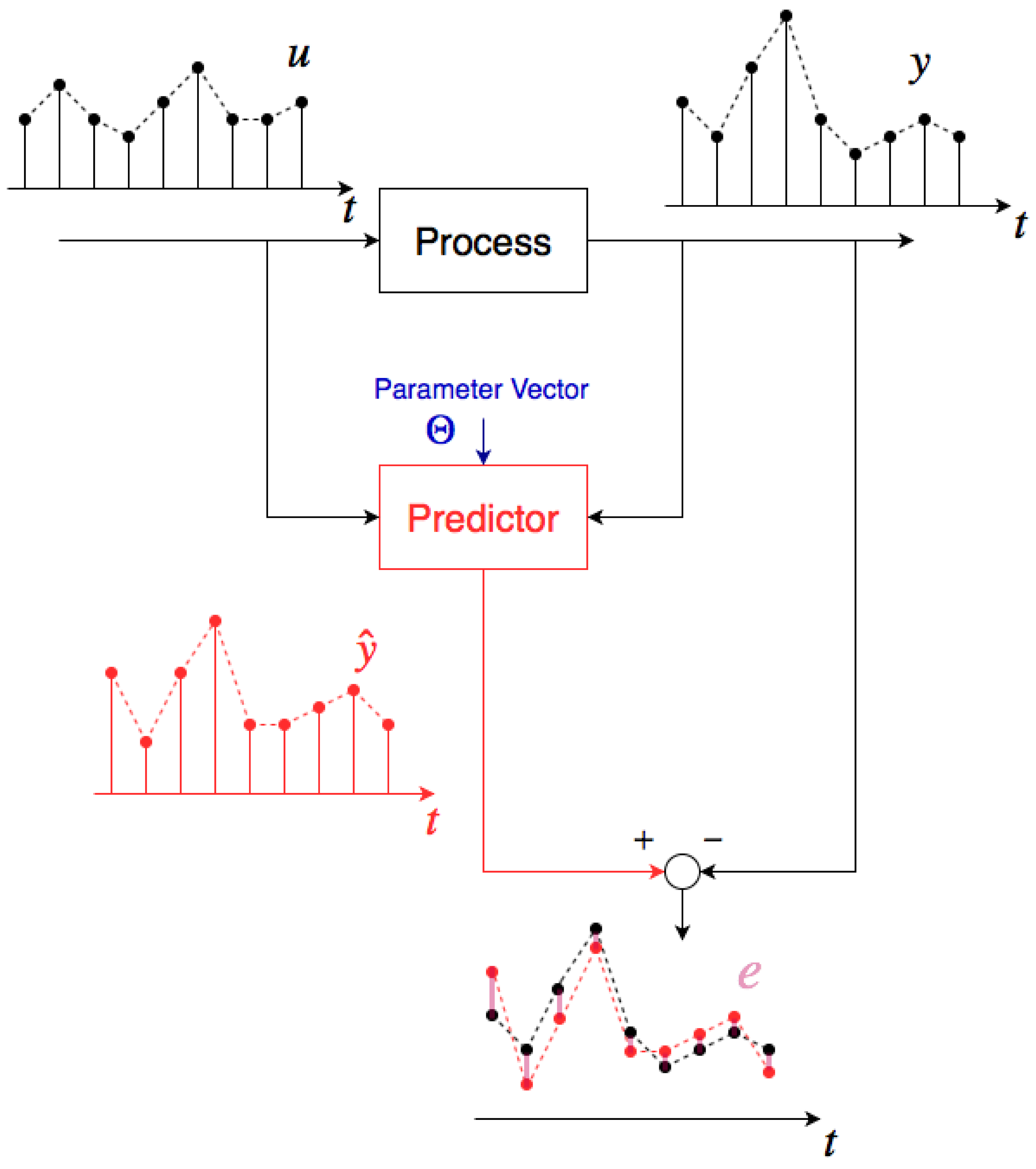

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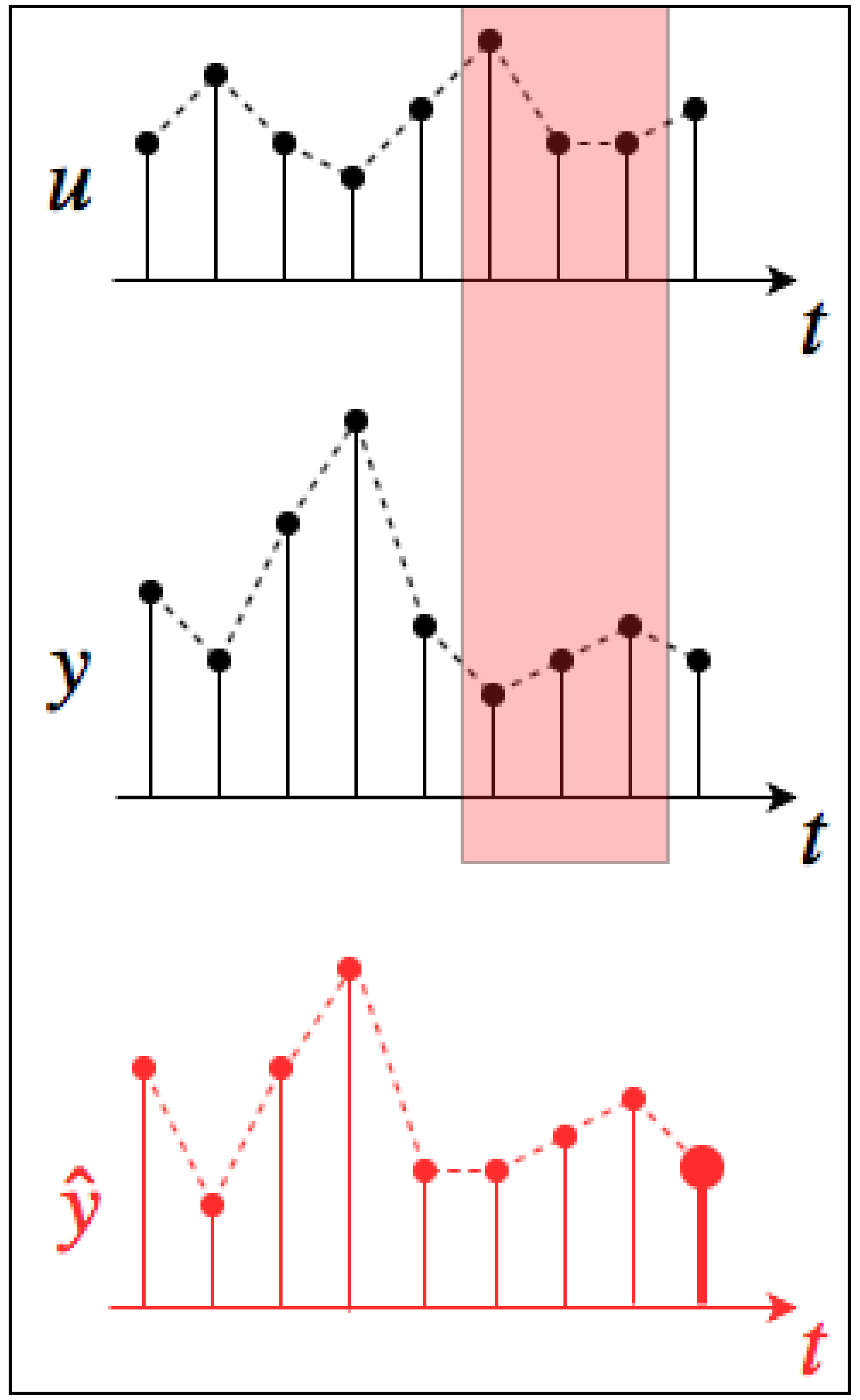
One-step-ahead Prediction



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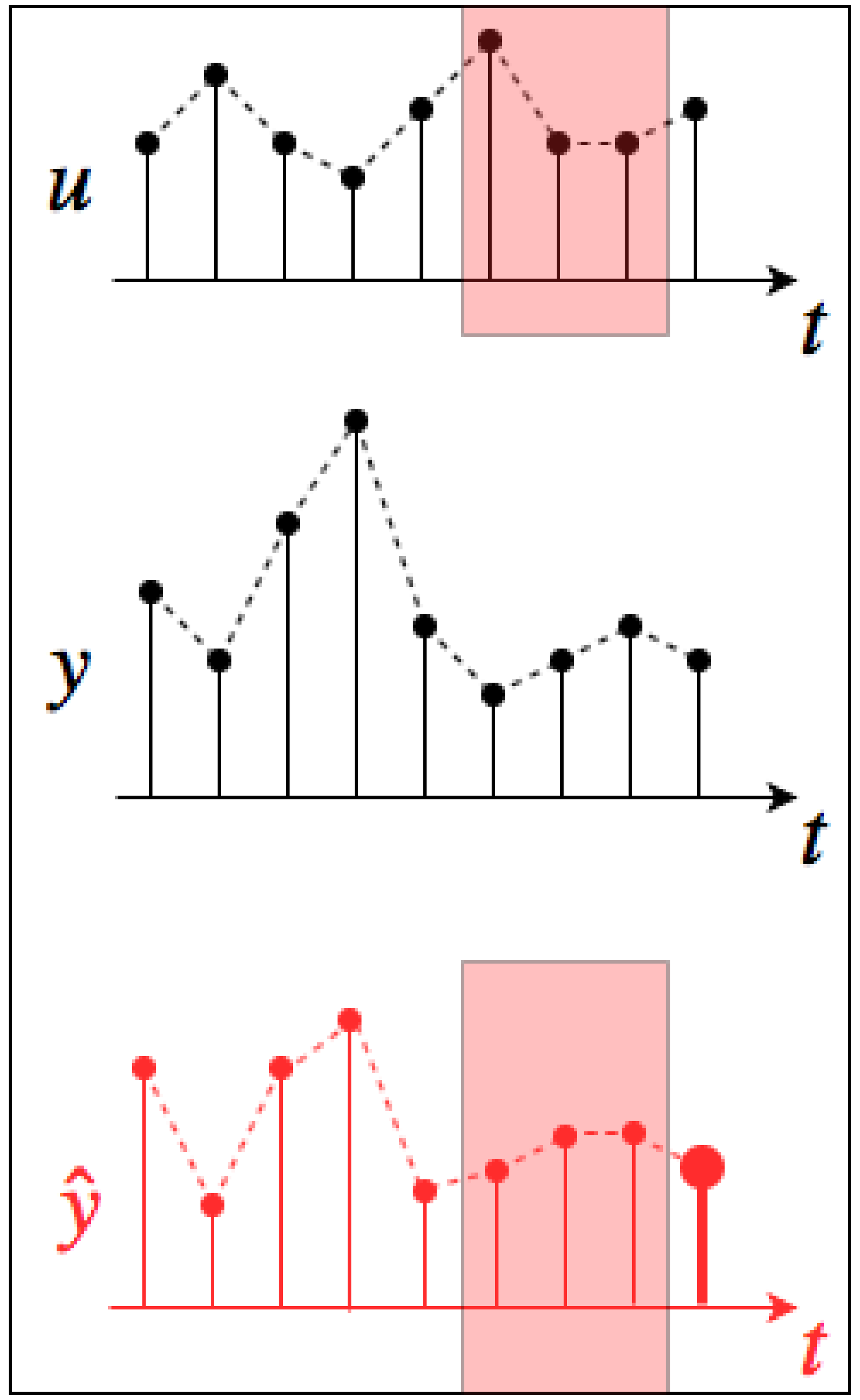
Free-run Simulation



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Prediction Error Methods

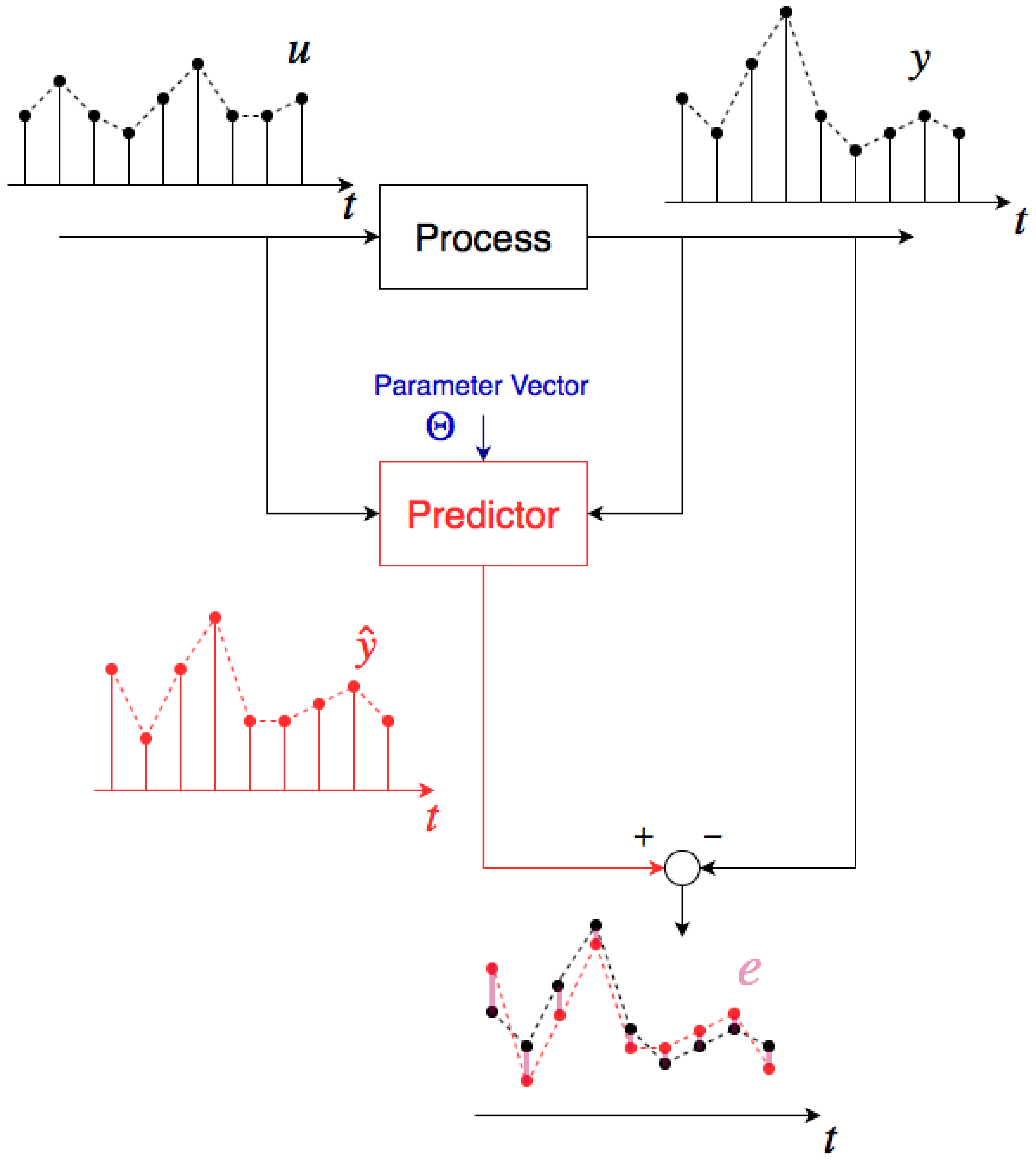

Figure 11: *Prediction error methods.* These methods estimate the parameters by minimizing the error between the optimal output prediction and the measured value

Difference Equation with Noise

$$y^*[k] = F(y[k-1], y[k-2], y[k-3], u[k-1], u[k-2], u[k-3]; \Theta^*) + v[k],$$
$$y[k] = y^*[k] + w[k].$$

NARX model

- NARX (nonlinear autoregressive with exogenous input) models follows from considering $w[k] = 0$ and $v[k]$ as white noise:

$$y^*[k] = F(\underline{y}_{[k]}, \underline{u}_{[k]}; \Theta^*) + v[k]$$
$$y[k] = y^*[k]. \quad (1)$$

- The optimal predictor for this situation is the **one-step-ahead prediction**.
- Parameters are estimated minimizing the sum of square errors between the **one-step-ahead prediction** and the measured values.

NOE model

- NOE (nonlinear output error) models follows from considering $v[k] = 0$ and $w[k]$ as white noise:

$$y^*[k] = F(\underline{y}_{[k]}, \underline{u}_{[k]}; \Theta^*)$$
$$y[k] = y^*[k] + w[k]. \quad (2)$$

- The optimal predictor for this situation is the **free-run simulation**.
- Parameters are estimated minimizing the sum of square errors between the **free-run simulation** and the measured values.

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Problem Statement

Consider the nonlinear system:

$$y^*[k] = \Theta_1 y^*[k-1] + \Theta_2 u[k-2] + \Theta_3 u^2[k-1] + \Theta_4 (y^*)^2[k-2] + \Theta_5$$
$$y[k] = y^*[k] + w[k]. \quad (3)$$

for which $\Theta = [0.5,\ 0.8,\ 1,\ -0.05,\ 0.5]^T$ and $w[k]$ is a white Gaussian output error.

Simulation Results

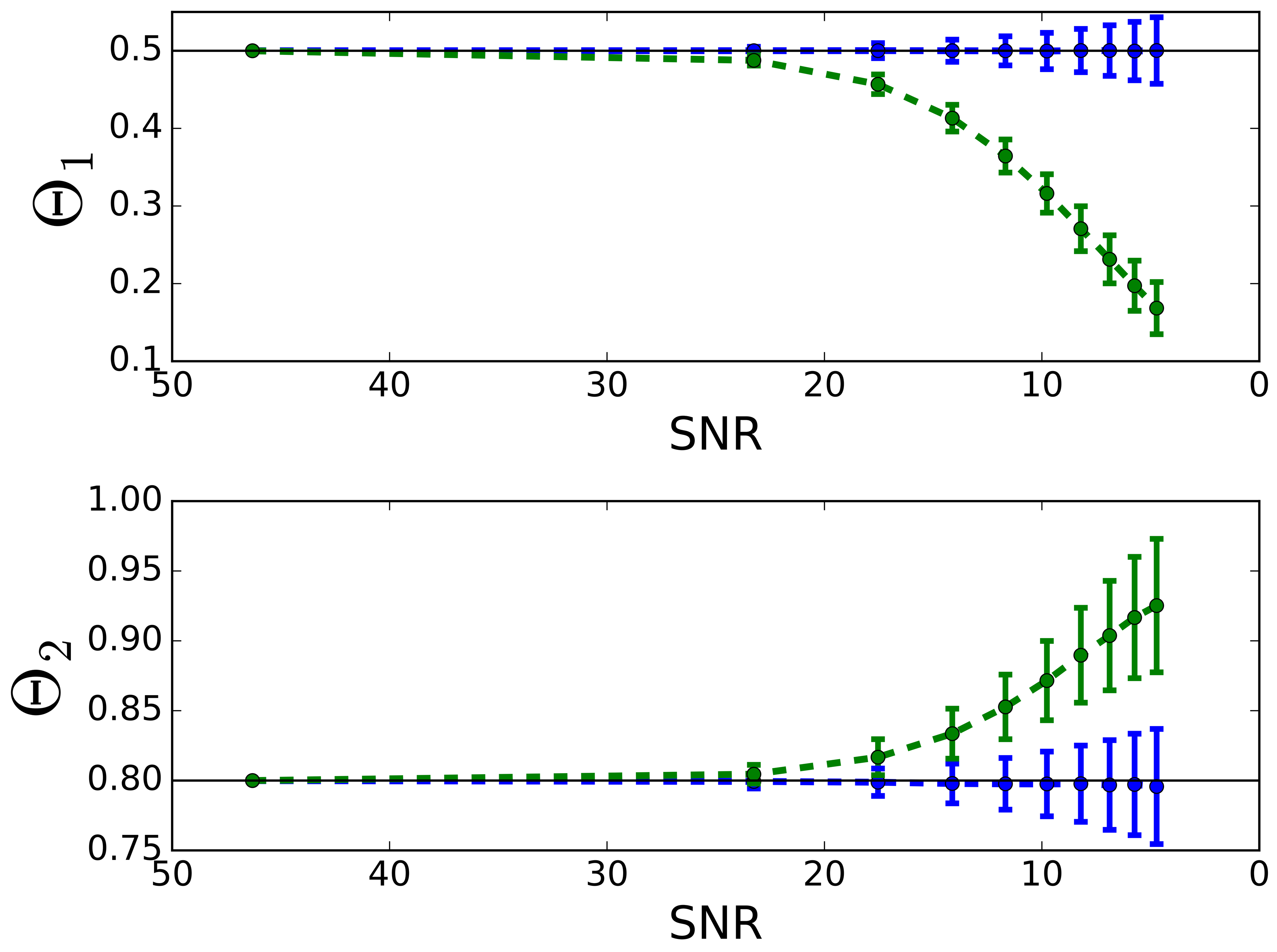

Figure 12: Expected values (dots) and standard deviation $\pm\sigma_{\hat{\Theta}}$ (bars) of parameter. Estimated using an NOE model (in blue) and an NARX model (in green). The true value, Θ_1, is represented as the horizontal line.

Generated Input and Output

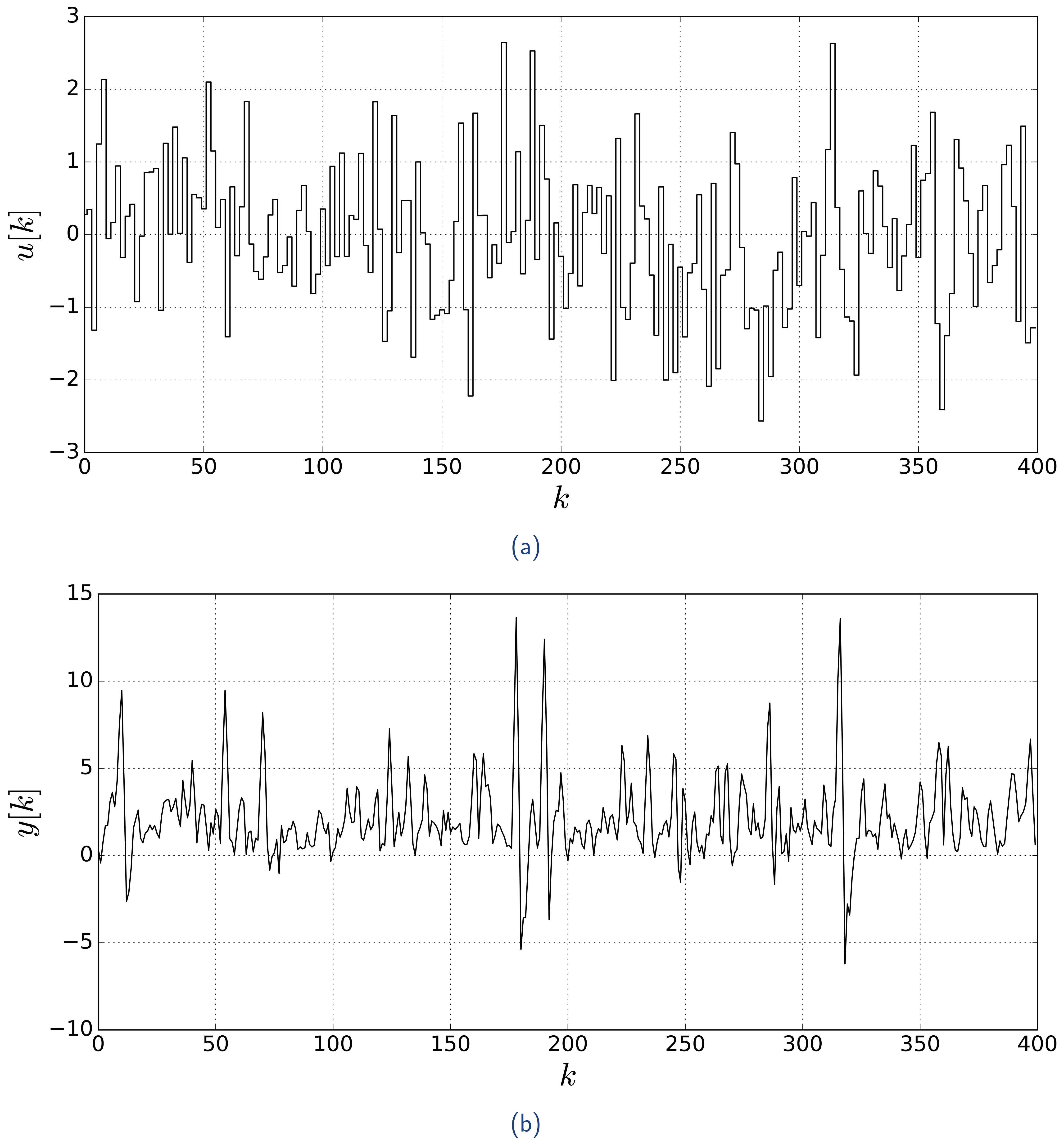

Figure 13: (a) Input; (b) Output.

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Single Shooting

Minimize free-run simulation error:

$$\min_{\Theta} \frac{1}{2}\|\mathrm{e_s}\|^2.$$

Multiple Shooting

- Suppose the dataset is subdivided into m_s smaller datasets;
- Let $\mathrm{e}_s^{(i)}$ be the error between the free-run simulation $\hat{y}^{(i)}$ and the correspondent measured output;
- $\mathrm{e_{ms}} = [(\mathrm{e}_s^{(1)})^T, \cdots, (\mathrm{e}_s^{(m_s)})^T]^T$;
- When the initial conditions of one simulation coincide with the end of the previous, we have: $\mathrm{e_{ms}} = \mathrm{e}_s$;
- Estimate NOE model parameters by solving the following constrained optimization problem:

$$\min_{\Phi} \frac{1}{2}\|\mathrm{e_{ms}}\|^2$$
$$\text{subject to: } \underline{\hat{\mathrm{y}}}^{(i)}[\text{end}] = \underline{\mathrm{y}}_0^{(i+1)},\ i = 1, \cdots, m_s - 1.$$

The variables for the optimization problem are both the parameter vector and the initial conditions:

$$\Phi = \left[\Theta^T\, \underline{\mathrm{y}}_0^{(1)T} \cdots \underline{\mathrm{y}}_0^{(m_s)T}\right]^T.$$

- Name is an analogy with ODE parameter estimation methods.

Ilustrative example

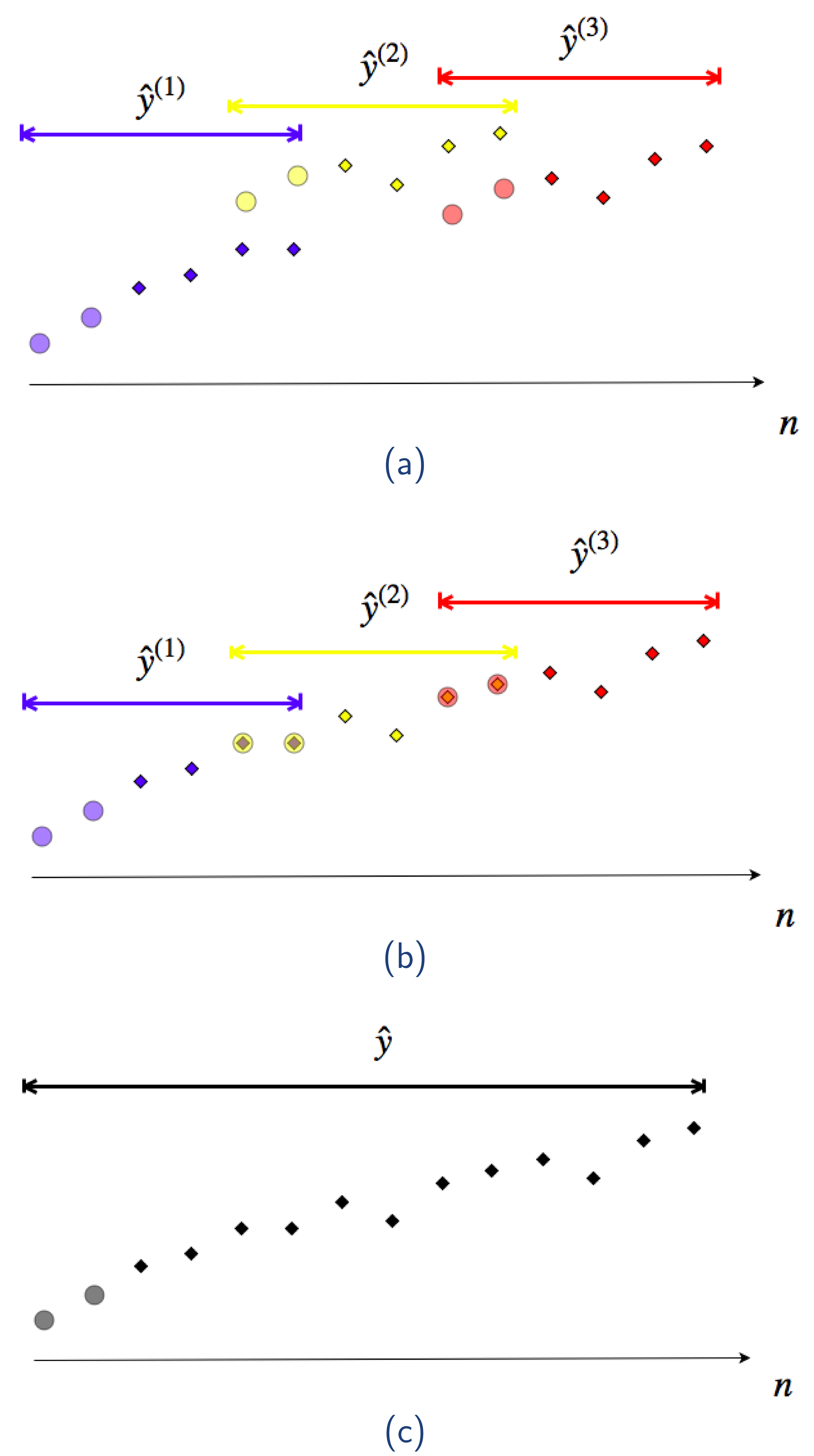

Figure 14: Three consecutive simulations $\hat{y}^{(i)},\ i = 1, 2, 3$ are indicated with different colors. The initial conditions are represented with circles ◦ and subsequent simulated values with diamonds ⋄.

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Problem Statement

A dataset with 300 samples for $\boldsymbol{\theta} = \mathbf{3.78}$ were generated using the logistic map:

$$\boldsymbol{y[k] = \theta y[k-1](1 - y[k-1])}.$$

A twofold approach is adopted to illustrate the limitations of single shooting method for this example: i) For the generated dataset, a slice of the optimization objective function is obtained by varying $\hat{\boldsymbol{\theta}}$; and, ii) for 100 initial guesses $\boldsymbol{\theta}^0$ chosen randomly (uniform) between 0 and 4 the parameter $\boldsymbol{\theta}$ parameter was estimated and the results displayed on a histogram.

Objective Function Slice

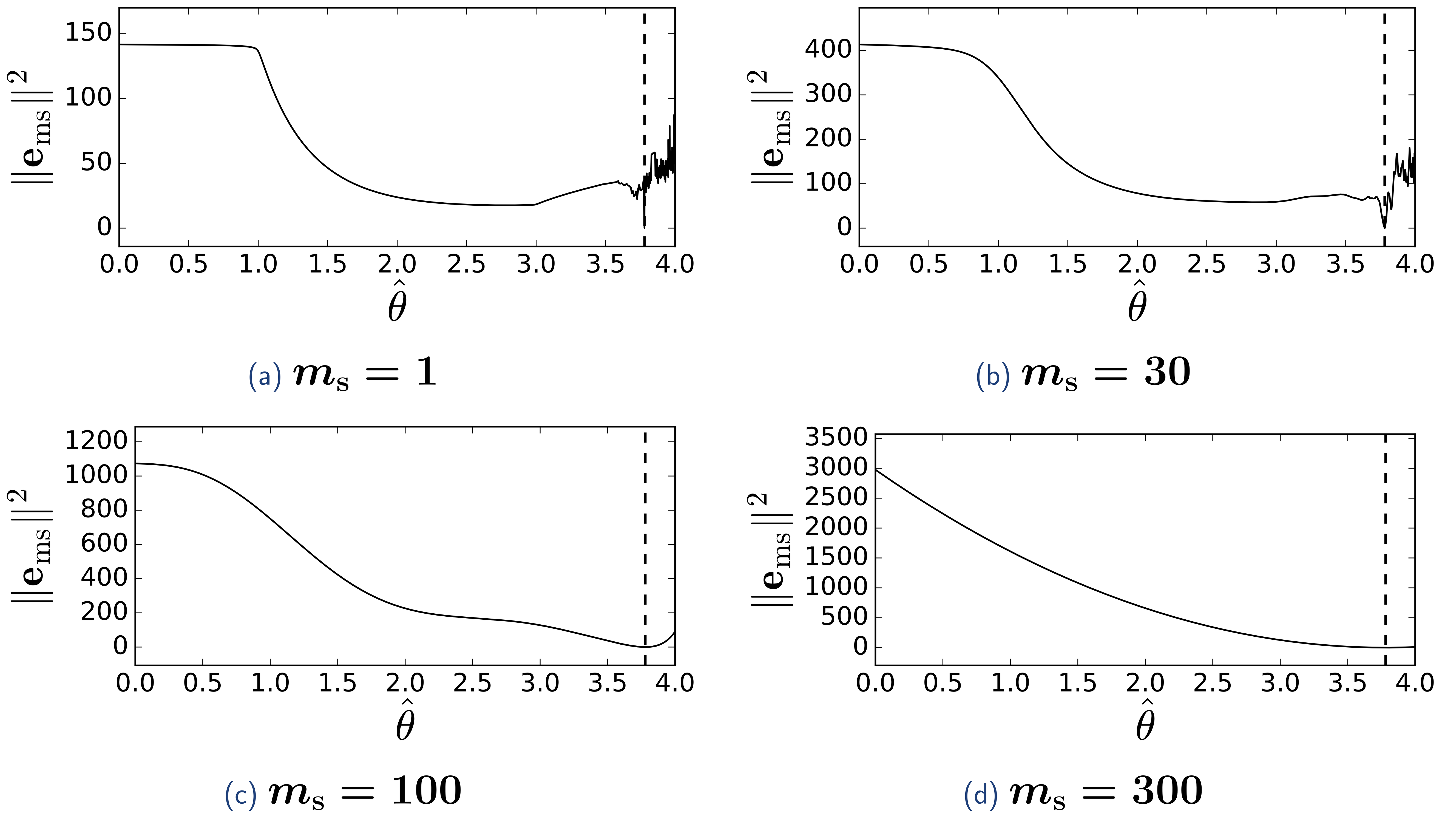

Figure 15: Shows $\|\mathrm{e}_{\mathrm{ms}}\|^2$ as a function of $\hat{\theta}$ for fixed initial conditions. The vertical dashed line (——) represents the true parameter value.

Convergence Histogram

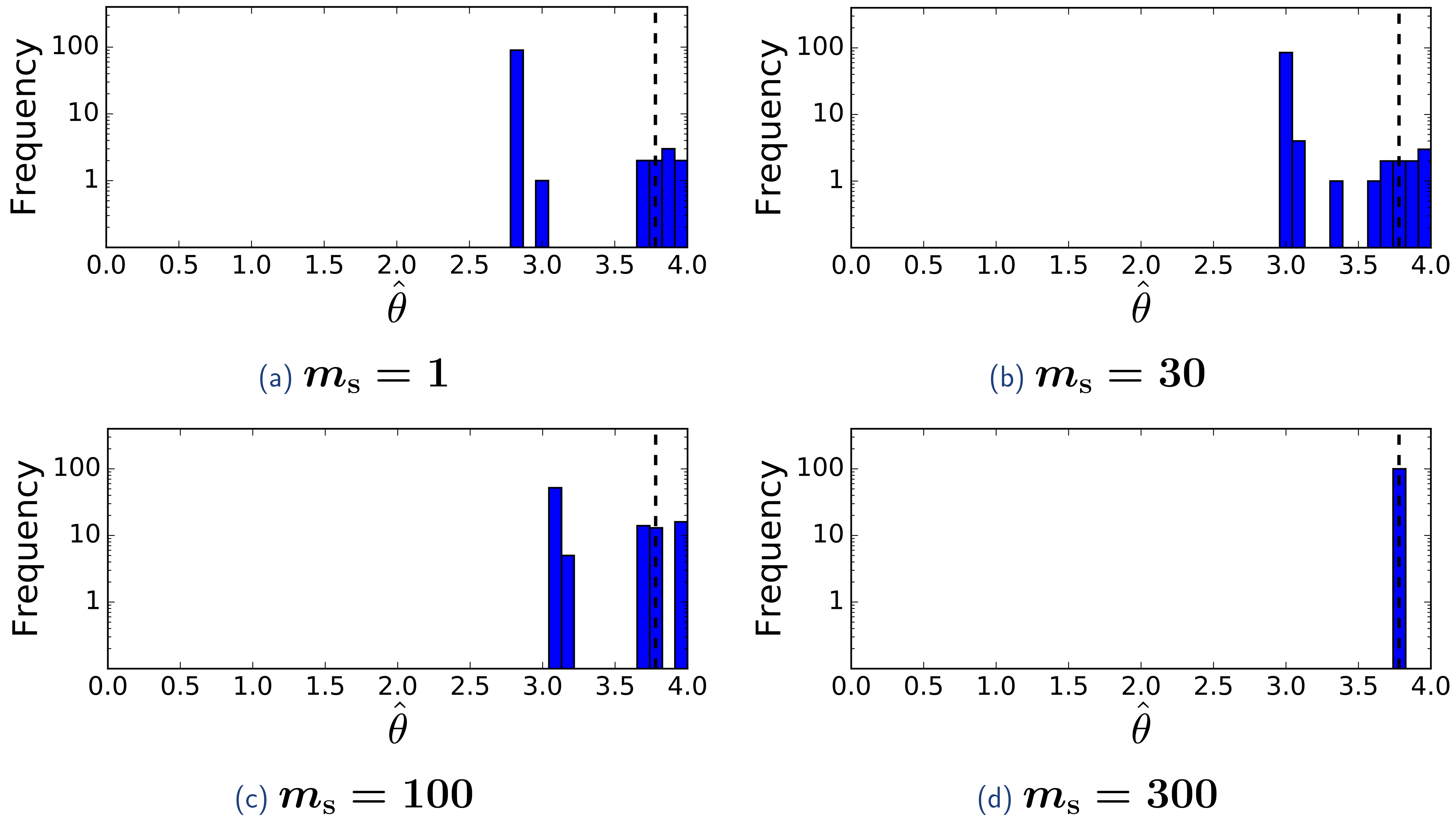

Figure 16: Histograms (in log scale) of the estimated parameter values to which the algorithm converged. The vertical dashed line (——) represents the true parameter value.